Stress Analysis and Optimization of Crankshafts Subject to Dynamic Loading

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FORWARD

The overall objective of this study was to evaluate and compare the fatigue performance of two competing manufacturing technologies for automotive crankshafts, namely forged steel and ductile cast iron. In addition, weight and cost reduction opportunities for optimization of the forged steel crankshaft were also investigated. The detailed results are presented in two reports. The first report deals with the fatigue performance and comparison of forged steel and ductile cast iron crankshafts. This second report deals with analyses of weight and cost reduction for optimization of the forged steel crankshaft.
ABSTRACT

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The main objective of this study was to investigate weight and cost reduction opportunities for a forged steel crankshaft. The need of load history in the FEM analysis necessitates performing a detailed dynamic load analysis. Therefore, this study consists of three major sections: (1) dynamic load analysis, (2) FEM and stress analysis, (3) optimization for weight and cost reduction.

In this study a dynamic simulation was conducted on two crankshafts, cast iron and forged steel, from similar single cylinder four stroke engines. Finite element analysis was performed to obtain the variation of stress magnitude at critical locations. The pressure-volume diagram was used to calculate the load boundary condition in dynamic simulation model, and other simulation inputs were taken from the engine specification chart. The dynamic analysis was done analytically and was verified by simulations in ADAMS which resulted in the load spectrum applied to crankpin bearing. This load was then applied to the FE model in ABAQUS, and boundary conditions were applied according to the engine mounting conditions. The analysis was done for different engine speeds and as a result, critical engine speed and critical region on the crankshafts were obtained. Stress variation over the engine cycle and the effect of torsional load in the analysis were investigated. Results from FE analysis were verified by strain gages attached to several locations on the forged steel crankshaft.
Results achieved from aforementioned analysis were used in optimization of the forged steel crankshaft. Geometry, material, and manufacturing processes were optimized considering different constraints, manufacturing feasibility, and cost. The optimization process included geometry changes compatible with the current engine, fillet rolling, and the use of microalloyed steel, resulting in 18% weight reduction, increased fatigue strength and reduced cost of the crankshaft, without changing connecting rod and/or engine block. A 26% weight reduction is also possible considering changes in the main bearings and the engine block.
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Nomenclature

\( a_p, a_{px} \)  Linear acceleration of piston, Linear acceleration of piston in X direction

\( a_{rx}, a_{ry} \)  X, Y components of linear acceleration of C.G. of connecting rod

\( d \)  Oil bore diameter located in the crankpin bearing

\( D, R_p \)  Diameter, radius of piston

\( e \)  Oil bore eccentricity distance from crankpin bearing center

\( F \)  Force magnitude applied on the crankpin bearing

\( F_{ax}, F_{ay} \)  X, Y components of reaction forces on the crankpin defined in non-rotating coordinate system

\( F_{px} \)  Reaction force at the piston pin end of connecting rod in the X direction

\( F_{sx}, F_{sy} \)  X, Y components of the load applied on the crankpin bearing described in a coordinate system, XY, attached to crankshaft

\( I \)  Moment of inertia of a cross section

\( I_{xx}, I_{yy}, I_{zz} \)  Moment of Inertia about X, Y, and Z axis located at the C.G. of connecting rod

\( I_{xy}, I_{yz}, I_{xz} \)  Moment of Inertia about XY, YZ, and XZ planes located at the C.G. of connecting rod

\( l \)  Location of bending neutral axis of a cross section

\( L_1 \)  Crank radius

\( L_2 \)  Connecting rod length

\( L_g \)  Distance of the C.G. of connecting rod from crankpin end center

\( m_p, m_r \)  Mass of piston assembly, connecting rod

\( P_0 \)  Normal pressure on the contact surface between connecting rod bearing and crankpin bearing
\( P_c \) Pressure in the cylinder applied on the top of the piston

\( r \) Crankpin bearing radius

\( r_g \) Location of C.G. of connecting rod

\( r_{gx}, r_{gy} \) X, Y components of location of C.G. of connecting rod

\( r_{px}, r_{py} \) X, Y components of piston pin location

\( r_p \) Location of piston pin with respect to the central axis of the crankshaft

\( t \) Crankpin bearing length

\( V_g \) Linear velocity of C.G. of connecting rod

\( V_{gx}, V_{gy} \) X, Y components of linear velocity of C.G. of connecting rod

\( V_{px} \) Linear velocity of piston in the X direction

\( \alpha_1, \alpha_2 \) Angular acceleration of crankshaft, connecting rod

\( \beta \) Connecting rod angle

\( \varphi \) Angle on the crankpin bearing measured from a line connecting crankpin center to central axis of crankshaft

\( \theta \) Crank angle

\( \sigma_c \) Critical bending stress with symmetric cross section (cylinder with a concentric hole)

\( \sigma_e \) Critical bending stress with non symmetric cross section (cylinder with an eccentric hole)

\( \sigma_{\text{von Mises}} \) Equivalent von Mises stress

\( \sigma_{xx}, \sigma_{yy}, \sigma_{zz} \) x, y, and z components of normal stresses

\( \sigma_{xy}, \sigma_{yx}, \sigma_{xz} \) xy, yz, and xz components of shear stresses

\( \omega_1, \omega_2 \) Angular velocity of crankshaft, connecting rod
1 Introduction

1.1 Background

Crankshaft is a large component with a complex geometry in the engine, which converts the reciprocating displacement of the piston to a rotary motion with a four link mechanism. Since the crankshaft experiences a large number of load cycles during its service life, fatigue performance and durability of this component has to be considered in the design process. Design developments have always been an important issue in the crankshaft production industry, in order to manufacture a less expensive component with the minimum weight possible and proper fatigue strength and other functional requirements. These improvements result in lighter and smaller engines with better fuel efficiency and higher power output.

This study was conducted on a single cylinder four stroke cycle engine. Two different crankshafts from similar engines were studied in this research. The finite element analysis was performed in four static steps for each crankshaft. Stresses from these analyses were used for superposition with regards to dynamic load applied to the crankshaft. Further analysis was performed on the forged steel crankshaft in order to optimize the weight and manufacturing cost. Figure 1.1 shows a typical picture of a crankshaft and the nomenclature used to define its different parts.
1.1.1 Function of Crankshafts in IC Engines

The crankshaft, connecting rod, and piston constitute a four bar slider-crank mechanism, which converts the sliding motion of the piston (slider in the mechanism) to a rotary motion. Since the rotation output is more practical and applicable for input to other devices, the concept design of an engine is that the output would be rotation. In addition, the linear displacement of an engine is not smooth, as the displacement is caused by the combustion of gas in the combustion chamber. Therefore, the displacement has sudden shocks and using this input for another device may cause damage to it. The concept of using crankshaft is to change these sudden displacements to a smooth rotary output, which is the input to many devices such as generators, pumps, and compressors. It should also be mentioned that the use of a flywheel helps in smoothing the shocks. Figure 1.2 shows the mounting of a crankshaft in an engine and Figure 1.3 shows the P-V diagram during an engine cycle for a four stroke cycle engine, where $V_d$ is the volume swept by the piston and $V_{bdc}$ is the volume of the cylinder when the piston is at the bottom dead centre (BDC).

1.1.2 Service Loads and Failures Experienced by Crankshafts

Crankshaft experiences large forces from gas combustion. This force is applied to the top of the piston and since the connecting rod connects the piston to the crankshaft, the force will be transmitted to the crankshaft. The magnitude of the force depends on many factors which consists of crank radius, connecting rod dimensions, weight of the connecting rod, piston, piston rings, and pin. Combustion and inertia forces acting on the
crankshaft cause two types of loading on the crankshaft structure; torsional load and bending load.

There are many sources of failure in the engine. They could be categorized as operating sources, mechanical sources, and repairing sources (Silva 2003). One of the most common crankshaft failures is fatigue at the fillet areas due to bending load caused by the combustion. Even with a soft case as journal bearing contact surface, in a crankshaft free of internal flaws one would still expect a bending or torsional fatigue crack to initiate at the pin surface, radius, or at the surface of an oil hole.

Due to the crankshaft geometry and engine mechanism, the crankshaft fillet experiences a large stress range during its service life. Figure 1.4 shows a crankshaft in the engine block from side view. In this figure it can be seen that at the moment of combustion the load from the piston is transmitted to the crankpin, causing a large bending moment on the entire geometry of the crankshaft. At the root of the fillet areas stress concentrations exist and these high stress range locations are the points where cyclic loads could cause fatigue crack initiation, leading to fracture.

1.2 Motivation and Objectives of the Study

This study was motivated by a need for a comparative study of forged steel and ductile cast iron crankshafts, which are the most commonly used manufacturing processes for an automotive crankshaft. In addition, it was desired to develop an optimized geometry, material, and manufacturing procedure which will reduce the weight of the forged steel component for fuel efficiency and reduce the manufacturing cost due to high volume production of this component.
This research was performed on crankshafts from single cylinder engines. However, since the basis of analysis are the same for multi-cylinder engines, the procedures used could be modified and implemented for crankshafts from other types of engines. Other studies on crankshafts from multi-cylinder engines are typically performed on a portion of the crankshaft consisting of two journal bearings and one crankpin bearing, which is similar to that a single cylinder engine. The only major difference in engines with different number of cylinders is the dynamic analysis of the loads applied to the component.

This study is a part of a project dealing with fatigue performance comparison and optimization of forged steel and ductile cast iron crankshafts. A comprehensive literature review about crankshaft durability, comparison of competing manufacturing processes and cost analysis was performed by Zoroufi and Fatemi (2005). Fatigue and durability assessment of the crankshafts used in this study were investigated by Williams and Fatemi (2007). They conducted experimental monotonic and cyclic tests on both materials using specimens taken from as forged and as cast crankshafts. Their investigation also included component fatigue tests and fatigue life predictions.

This study is concerned with identifying the critical location of fatigue failure caused by engine loads and calculating the operating stress range at this location (Montazersadgh and Fatemi, 2007). In addition, considering component geometry and according to the applied loads during service life of the crankshaft, it was desired to optimize the weight and manufacturing cost of the forged steel crankshaft, while maintaining or improving its fatigue performance (Williams et al., 2007).
Chapter 2 discusses forging and casting manufacturing processes of crankshaft production. In this chapter, these two competing manufacturing processes are explained and compared. This chapter also presents a literature survey performed on previous studies. Chapter 3 discusses dynamic load analysis of the crankshaft, which consists of load-history applied to the crankshaft bearings during its service life. Stress analysis and FEA, according to dynamic loading of the crankshaft, is investigated and presented in Chapter 4. This is followed by the optimization study of the forged steel crankshaft in Chapter 5. The optimization process started with geometry optimization and was completed by considering modifications to the manufacturing process and using alternative materials.
Figure 1.1 Typical crankshaft with main journals that support the crankshaft in the engine block. Rod journals are offset from the crankshaft centerline (Halderman and Mitchell, 2001).

Figure 1.2 Exploded view of a single cylinder engine showing the crankshaft mounting in the engine (http://www.deere.com/, 2005).
Figure 1.3  P-V diagram at constant delivery ratio. Curve 5 is for 900 rev/min, curve 6 for 1200 rev/min, curve 7 for 1500 rev/min, and curve 8 for 1800 rev/min (Ferguson, 1986).

Figure 1.4  Side view of the engine block at the time of combustion (http://www.wikipedia.org/, 2007).
2 Literature Review

An extensive literature review on crankshafts was performed by Zoroufi and Fatemi (2005). Their study presents a literature survey focused on fatigue performance evaluation and comparisons of forged steel and ductile cast iron crankshafts. In their study, crankshaft specifications, operation conditions, and various failure sources are discussed. Their survey included a review of the effect of influential parameters such as residual stress on fatigue behavior and methods of inducing compressive residual stress in crankshafts. The common crankshaft material and manufacturing process technologies in use were compared with regards to their durability performance. This was followed by a discussion of durability assessment procedures used for crankshafts, as well as bench testing and experimental techniques. In their literature review, geometry optimization of crankshafts, cost analysis and potential cost saving opportunities are also briefly discussed.

In this chapter, material and manufacturing processes of crankshafts are discussed and a brief literature review related to this topic is presented. Since this study is mostly about dynamic load analysis, finite element modeling, and optimization, research articles concerning these issues are also summarized and discussed in this chapter.
2.1 Materials and Manufacturing Processes

The major crankshaft material competitors currently used in industry are forged steel, and cast iron. Comparison of the performance of these materials with respect to static, cyclic, and impact loading are of great interest to the automotive industry. A comprehensive comparison of manufacturing processes with respect to mechanical properties, manufacturing aspects, and finished cost for crankshafts has been conducted by Zoroufi and Fatemi (2005).

This Section discusses forging and casting processes as the two competing manufacturing processes in crankshaft production industry. Influencing parameters in both processes are detailed. Finally, the forged steel and the cast iron products are compared in terms of material properties and manufacturing processes.

2.1.1 Forging Process and the Influencing Parameters

Forging is the term for shaping metal by plastic deformation. Cold forging is done at low temperatures, while conventional hot forging is done at high temperatures, which makes metal easier to shape. Cold forgings are various forging processes conducted at near ambient temperatures, such as bending, cold drawing, cold heading, coining, and extrusion to produce metal components to close tolerances and net shape (http://www.jobshop.com/, 2007). Warm forging is a modification of the cold forging process where the workpiece is heated to a temperature significantly below the typical hot forging temperature, ranging from 500º C to 750º C (http://www.forging.org/, 2007). Compared with cold forging, warm forging has the potential advantages of reduced tooling loads, reduced press loads, increased steel ductility, elimination of need to anneal
prior to forging, and favorable as-forged properties that can eliminate heat treatment (http://www.qcforge.net/, 2007). The use of the lower temperatures in cold and warm forging processes provides the advantages of reducing and even substantially eliminating the harmful scale or oxide growth on the component as well as enabling the component to be produced to a high dimensional accuracy (Koike and Matsui, 2001). Despite these advantages, cold and warm forging processes have the limitations of close tolerance and net shape of the final component with the workpiece. Hot forging is the plastic deformation of metal at a temperature and strain rate such that recrystallization occurs simultaneously with deformation, thus avoiding strain hardening (http://www.jobshop.com/, 2007). Since crankshafts have complex geometries, warm and cold forging of the component is not possible. Therefore, crankshafts are manufactured using the hot forging process.

In impression (or closed die) hot forging two or more dies are moved toward each other to form a metal billet, at a suitable temperature, in a shape determined by the die impressions. These processes are capable of producing components of high quality at moderate cost. Forgings offer a high strength to weight ratio, toughness, and resistance to impact and fatigue, which are important factors in crankshaft performance.

In closed die forging, a material must satisfy two basic requirements: (a) The material strength (or flow stress) must be low so that die pressures can be kept within the capabilities of practical die materials and constructions, and (b) The capability of the material to deform without failure (forgeability) must be sufficient to allow the desired amount of deformation.
By convention, impression and closed die forging are considered to be hot working operations. In most practical hot forging operations, the temperature of the work-piece material is higher than that of the dies. The metal flow and die filling are largely determined by (a) the forging material’s resistance to flow and ability to flow, i.e. its flow stress and forgeability, (b) the friction and cooling effects at the material interface and (c) the complexity of the forging shape. A multi-cylinder crankshaft is considered to have a complex geometry, which necessitates proper workpiece and die design according to material forgeability and friction to have the desired geometry (Altan et al., 1983).

**Lubrication**

In hot forging, in addition to lubrication effects, the effects of die chilling or heat transfer from the host material to the colder dies must be considered. Therefore, values of the friction factor, or coefficient of friction, obtained under certain forging conditions may not be applicable under other conditions. For example, for a given lubricant, friction data obtained in hydraulic press forging cannot be useful in mechanical press or hammer forging, even if the die and billet temperatures are comparable (Altan et al., 1983).

**Shape complexity in forging**

The main objective of forging process design is to ensure adequate flow of the metal in the dies so that the desired finish part geometry can be obtained without any external or internal defects. Metal flow is greatly influenced by part or die geometry. Often, several operations are needed to achieve gradual flow of the metal from an initially simple shape (cylinder or round cornered square billet) into the more complex shape of the final forging (Altan et al., 1983).
Schematic initial workpiece and lower forging die for a four cylinder crankshaft is shown in Figure 2.1. The as-forged crankshaft with flashes is shown in Figure 2.2. As could be seen, the geometry complexity requires extensive analysis on the forging piece and die, in order to have proper material flow to fill all cavities in the die. Improper workpiece design will result in a deficient geometry as shown in Figure 2.3 (Shamasundar et al., 2003).

**Heat treatment**

All hot forged parts receive a certain amount of heat treatment in the process of being forged and, thereafter, may be used without additional heat treatment. For maximum usefulness, however, many forgings are heat treated one or more times before being put into service. For instance, bearing sections and fillet areas on crankshafts are heat treated in order to improve fatigue and wear properties of the material at these certain locations.

Usually forgings are heat treated before and after their machining. The purpose of the initial treatment is to secure uniform structure of the metal and contribute to ease of machining of the forged part. The final treatment makes it possible to use the finished forgings for the service intended. For example, forged tools must be hard and tough, consequently, they must receive final hardening and tempering treatments (http://www.sme.org/, 2007). Today, microalloy steels are widely used for manufacturing forged steel crankshafts. The microalloyed steel can effectively eliminate the need for additional heat-treating, reducing the cost of forgings by 5% to 10% to the final cost (http://www.steel.org/, 2007).
Straightening and coining

When the flash is trimmed from the drop forging, the shape may become distorted, which is common in forged crankshafts because of geometry section changes and non-uniform cooling during forging process. Correction of this condition may be necessary. Correction to a certain degree may be accomplished by hammering the distorted forging in a special re-striking die. The correction is made while the forging cools. Other re-striking operations, called coining, are conducted on powerful and accurate presses after the forgings have cooled to room temperature. The forgings are brought to the correct size and shape in these presses, and final machining operations ordinarily performed are either entirely or partially eliminated (http://www.engineersedge.com/, 2007).

Microalloy steels provide more consistent mechanical properties, improved steelmaking and metal forming processes such as rolling, and reduced surface flaws with less distortion than heat-treated forged steel. This reduces the need for straightening in the forging process and increases the machinability of the forged steel (http://www.steel.org/, 2007).

2.1.2 Casting Process and the Influencing Parameters

Casting is a manufacturing process by which a molten material such as metal or plastic is introduced into a mold, allowed to solidify within the mold, and then ejected or broken out to make a fabricated part. Casting is used for making parts of complex shape, such as crankshafts, that would be difficult or uneconomical to make by other methods (such as machining from solid material).
Sand-mold casting is adaptable to a very wide range of alloys, shapes, sizes, and production quantities. Hollow shapes can be produced in these castings through the use of cores. Sand-mold casting is by far the most common casting process used in the industry; some estimates are that as many as 90% of industrial castings use the sand-mold casting process [http://www.makinamuhendisi.com/, 2007].

**Green sand**

Green sand refers to the fact that water is added to activate the clay binder. Green sand-mold casting involves mixing sand with a suitable clay binder (usually a bentonite clay) and other additives, and packing the sand mixture tightly around a pattern that is constructed from the part design, but the pattern is not an exact replica of the part since various dimensional allowances must be made to accommodate certain physical effects. For most metals and most sizes and shapes of casting, green sand molding is the most economical of all the molding processes.

The moisture present in green sand produces steam when contacted by hot metal. Inability of the steam and other gases to escape causes problems with some casting designs, and blowhole damage results. The dimensional accuracy of green sand casting is limited. Even with small castings, it is seldom that dimensions can be held closer than ±0.5 millimeter [http://www.makinamuhendisi.com/, 2007].

**Dry sand molds**

Dry sand molding is a sand casting process using a mold made of greensand which is dried in an oven to remove the moisture and increase its strength. The absence of moisture eliminates the formation of water vapor and reduces the type of casting defects that are due to gas formation. The cost of heat, the time required for drying the
mold, and the difficulty of handling heavy molds without damage make the process expensive compared to green sand molding, and it is used mostly when steam formation from the moisture present would be a serious problem (http://www.castsolutions.com/, 2007).

**Skin drying**

Most of the benefits of dry sand molds can be obtained by skin drying molds to depths from a fraction of an inch to an inch. With the mold open, the inside surfaces are subjected to heat from torches, radiant lamps, hot dry air, or electric heating elements to form a dry insulating skin around the mold cavity. Skin-dried molds can be stored only for short periods of time before pouring, since the water in the main body of the mold will redistribute itself and re-moisturize the inside skin (http://www.castsolutions.com/, 2007).

### 2.1.3 Comparison of Forging and Casting Processes

To some extend, forging and casting are competitive, even where different materials are involved with each process. As a general rule, the tooling investment is higher for forging than for casting. Thus, the use of forging tends to be restricted to applications in which the higher material properties of steel compare to cast iron or the higher properties of wrought steel compared to cast steel can be made use of in the design. Because forgings compete best in high strength applications, most producers take particular care in raw material selection and inspection. In many cases, either forging or castings may have adequate properties, and one process has no universal economic advantage over the other.

Some characteristics of castings as compared to forgings are as follows:
• Castings cannot obtain the strengthening effects of hot and cold working. Forging surpasses casting in predictable strength properties, producing assured superior strength consistently.

• A casting has neither grain flow nor directional strength and the process cannot prevent formation of certain metallurgical defects. Pre-working forge stock produces a grain flow oriented in directions requiring maximum strength. Dendritic structures, alloy segregations and like imperfections are refined in forging.

• Casting defects occur in a variety of forms. Because hot working refines grain pattern and imparts high strength, ductility and resistance properties, forged products are more reliable. And they are manufactured without the added costs for tighter process controls and inspection that are required for casting.

• Castings require close control of melting and cooling processes because alloy segregation may occur. This results in non-uniform heat-treatment response that can affect straightness of finished parts. Forgings respond more predictably to heat treatment and offer better dimensional stability.

• Some castings, such as special performance castings, require expensive materials and process controls, and longer lead times. Open-die and ring rolling are examples of forging processes that adapt to various production run lengths and enable shortened lead times.

In spite of the aforementioned advantages of forgings over castings, castings may be an economical alternative, depending on part functionality requirements, production volume, and other considerations.
2.2 Operating Conditions and Failure of Crankshafts

Crankshaft is one of the largest components in the internal combustion engine that has a complex geometry consisting of cylinders as bearings and plates as the crank webs. Geometry section changes in the crankshaft cause stress concentration at fillet areas where bearings are connected to the crank webs. In addition, this component experiences both torsional and bending load during its service life. Therefore, fillet areas are locations that experience the most critical stresses during the service life of the crankshaft. As a result, these locations are main sections of fatigue failure of the component. The size of a crankshaft depends on the number of cylinders and horsepower output of the engine. The size of the crankshaft could range from 3.2 kg for a single cylinder engine with the output power of 12 hp, to 300 tons for a fourteen cylinder diesel engine with the output power of 108,920 hp.

In an internal combustion engine, two load sources apply force on the crankshaft. The load applied by combustion in the combustion chamber to the piston is transmitted to the crankpin bearing by a four bar slider-crank mechanism. This is the main source of loading in the engine. The other load source is due to dynamic nature of the mechanism. Since the engine operates at high speeds, the centrifugal forces are present at different rotating components such as connecting rods. These load sources apply both torsional and bending load on the crankshaft.

Silva (2003) classifies the cause of journal bearing failure or damage (jagged journals) to three possible sources; “(a) operating sources such as oil absence on carter, defective lubrication of journals, high operating oil temperature, improper use of the engine (over-revving); (b) mechanical sources such as misalignments of the crankshaft on
the assembling, improper journal bearings (wrong size), no control on the clearance size between journals and bearings, crankshaft vibration; (c) repairing sources such as misalignments of the journals (due to improper grinding), misalignments of the crankshaft (due to improper alignment of the crankshaft), high stress concentrations (due to improper grinding at the radius on both sides of the journals), high surface roughness (due to improper grinding, origination of wearing), improper welding or nitration, straightening operation, defective grinding.” Details of cracks on the journal bearing surfaces studied by Silva are shown in Figure 2.4.

Another common failure in the crankshafts is mechanical crack nucleation at the fillet radius of journal bearings. Different criteria are used in identifying the crack in the fillet area. Spiteri et al. (2005) studied the relationship of different failure modes such as surface cracks, stiffness change, and two-piece failures on different crankshafts. The crankshafts in their study consisted of rolled, ductile, and cast iron crankshafts. Using the resonant bending test in their study they showed that an accelerating resonant shift is a valid failure criterion and the results have good agreement with two piece failure. In addition, the fatigue strength obtained from the two-piece failure criterion is significantly higher than the fatigue strength derived from the surface crack failure criterion. The four-bubble failure criterion cracks or those found in post process Magnaglow inspection did not propagate and result in failure. These cracks formed and remained on the fillet surface. Figure 2.5 shows a V6 sample failed with a 2 Hz resonance shift with a 9.5 mm track in Spiteri et al. studies.

Another crack detection method was introduced by Baxter (1993). He studied crack detection using a modified version of the gel electrode technique. In this technique
the critical location of the crankshaft was covered by a thin (~0.5 µm) polymer film. After the fatigue test, the surface was inspected with a gel electrode probe. Applying a voltage between the electrode as cathode and the crankshaft as anode, induced a current to flow to any fatigue sites in the contact area. Information was displayed qualitatively as a direct image of the fatigue sites and quantitatively as the spatial distribution of charge flow to the surface. This technique could identify both the primary fatigue cracks and a distribution of secondary sites of less severe fatigue damage. The most useful aspect of this study is that the ELPO film can be applied before or after the fatigue test, and in both cases, the gel electrode technique is successful at detecting fatigue damage. Figure 2.6 shows the photograph of the test piece after fracture at the fillet location. As can be seen, a fatigue crack of length 2.2 cm exists along the edge of the fillet, which the markings from this technique clearly identify.

2.3 Dynamic Load Determination and Analysis

Jenson (1970) performed an experimental study to determine the load applied to a V8 crankshaft. The load determination in this study started with the selection of the crankshaft sections to be investigated. The critical sections are shown on the V8 crankshaft in Figure 2.7. In order to measure the bending and torsion loads applied to each section of the crankshaft, bending and torsion strain gage bridges were mounted in pairs. Figure 2.8 shows the details of strain gage bridges mounted on section No. 4 in Figure 2.7. After mounting the strain gages, the crankshaft was carefully assembled in the engine and then the engine was completely assembled and installed on a dynamometer stand. The loads at several speed increments were recorded to capture peak bending and
torsion loads, which in general do not occur at the same crankshaft angle, as shown in Figure 2.9.

Henry et al. (1992) implemented the dynamic load in their FEM model with consideration of internal centrifugal, external bearing, and torsional dynamic loads. In their study, internal loads were calculated by assuming constant FEM mass forces. Therefore, for any engine speed, the resulting displacements were calculated only once. Considering the classical gas and inertia forces acting on the crankpin bearings result in external bearing loads. A statistically determinate method or an indeterminate one was used to calculate the journal bearings reactions considering the clearances and engine block compliance. And finally a classical mass-spring model with harmonic response was created in order to calculate the dynamic twisting moments. The result was the crank internal moment at each throw and throughout the cycle. Only one torsional displacement calculation was performed per throw, and the displacements at each cycle step were scalar multiples of these results.

An analytical investigation on bending vibrations was done for a V6 engine by Mourelatos (1995). He used a crankshaft system model (CRANKSYM) to analytically verify a vibration problem related to the flywheel for the mentioned crankshaft. As described in their study, CRANKSYM could perform an analysis considering the crankshaft structural dynamics, main bearing hydrodynamics and engine block flexibility. The program considers the gyroscopic effect of the flywheel, loads applied on the belt, crankshaft bent and block misboring, and the anisotropy of the block flexibility as seen from a rotating crankshaft. This program could also calculate the dynamic stress history on the crankshaft during the whole engine cycle. The CRANKSYM requires a finite
element mesh of the whole crankshaft assembly. The output of CRANKSYM includes
the natural frequencies of the crankshaft-flywheel system, the bearing loads with respect
to an engine fixed coordinate system, and the axial displacement of a point on the outer
perimeter of the flywheel. In this program the crankshaft structural analysis predicts the
crankshaft dynamic response based on the finite element method. Figure 2.10 shows the
finite element mesh for the crankshaft-flywheel assembly used in the CRANKSYM
program.

The classical model of crankshaft vibration was used in Payer et al. (1995) studies
to determine the time dependent displacement-vector \( x \) of the crankshaft for the
calculation of its transient stress behavior. The following equation was used in their
studies:

\[
[M]\ddot{x} + [D]\dot{x} + [C]x = F(t)
\]

(2.1)

where \([M]\), \([D]\) and \([C]\) are time dependent nonlinear mass matrix, damping matrix and
stiffness matrix, respectively, and the displacement vector as \( x \), velocity vector as \( \dot{x} \), and
acceleration vector as \( \ddot{x} \), and the time dependent force vectors as \( F(t) \). In this equation,
by knowing the displacements, velocities, and accelerations at time \( t \), the Newmark-Beta-
method could be used to calculate the solution for the time \( t + dt \). The excitation forces
acting at the crank pins are calculated from superposition of the time dependent gas
forces and the time dependent rotating mass forces of the pistons and connecting rods
considering the combustion order in the engine. This calculation results in the time
dependent deformation behavior of the crankshaft and the time dependent stress and
deformation behavior of the main bearing walls.
Prakash et al. (1998) used the advantages of both the classical method and finite element technique in their studies to design crankshafts. They used the classical method in order to calculate the initial and the approximate results. Based on this method a program was developed, TVAL, which quickly gave the natural frequencies, critical modes, displacements and stresses. In order to obtain more accurate results and to evaluate the results from the program, they created the finite element model of the crankshaft. Time varying radial and tangential forces acting on the crankpin were derived from the cylinder pressures. The displacements and stresses were calculated using the superposition method. They used an engine load duty cycle corresponding to automotive applications for generating load histories for accelerated fatigue testing.

### 2.4 Finite Element Modeling

Since the crankshaft has a complex geometry for analysis, finite element models have been considered to give an accurate and reasonable solution whenever laboratory testing is not available.

Uchida and Hara (1984) used a single throw FEM model shown in Figure 2.11 in the extrapolation of the experimental equation in their study. In their study the web thickness of a 60° V-6 crankshaft was reduced while maintaining its fatigue performance and durability under turbo-charged gas pressure. In the study of the 60° V-6 engine crankshaft dimensions, the stress evaluation of the crankpin fillet part was extremely critical since, to reduce the crank’s total length, it was necessary to reduce the thickness of the web between main journal and crankpin as much as possible while maintaining its
strength. FEM was used to estimate the stress concentration factor at thicknesses where confirmed calculation was not available.

A crankshaft durability assessment program based on three dimensional mechanical analyses was developed by RENAULT and was used by Henry et al. (1992) to predict the durability and calculate the fatigue performance of crankshafts. In the program, the stress calculations involved initial 3D FEM analysis with a coarse mesh for the whole crankshaft geometry as shown in Figure 2.12 and were followed by a local BEM fillet zoom technique for the fillet areas in order to accurately calculate the stress concentration factors. In their durability assessment program, they had chosen a 3D numerical approach. The coarse mesh is used in the whole crankshaft model to avoid cut plane boundary condition approximations. The resulting displacements were then applied to a local small sized BEM fillet model. The resulting BEM fillet stress state was biaxial on the surface, unlike corresponding FEM results. This two-stage FEM/BEM stress calculation method was compared to several other numerical calculations and several crankshaft geometries in their study, which showed increased precision, resulting in improved initial design of a crankshaft.

A theoretical study followed by experimental results was conducted by Guagliano et al. (1993) to calculate the stress concentration factor in a diesel engine crankshaft. They conducted experimental tests by mounting strain gages at high stress concentration areas (crank fillet). A three dimensional model of the crankshaft was generated and numerical calculation was performed according to linear-elastic properties of the material and different loading conditions. Good agreement between experimental results and numerical results indicated the accuracy of stress concentration factor calculation. In
addition, a bi-dimensional model of the crankshaft was constructed numerically and experimentally, in order to quickly evaluate the stress concentration factor. Considering similar boundary conditions and loading situation for both three dimensional and bi-dimensional models, resulted in identical stress concentration factors. They concluded that a bi-dimensional model could simply and efficiently determine the stress concentration factor.

An artificial neural network was developed by Shiomi and Watanabe (1995) in order to calculate the stress concentration factor at the crankpin fillet area from dimensional characteristics of the crankshaft. An artificial neural network was used to calculate the stress concentration factor based on mathematical approximations from a database which contained geometry properties of the crankshaft as well as stress concentration factors. In order to construct the database, finite element model of crankshafts with different geometries were created. Stress concentration factors were obtained from these models and results were inserted in the database. The finite element model results in accurate values for stress concentration factor and eliminates inevitable measurement errors due to experiments. A large number of mathematical functions had to be solved in order to create the approximation function, but after having the final equation, the stress concentration could be easily calculated for a given crankshaft geometry. The structure of the network applied to crankshaft geometry is shown in Figure 2.13.

A nonlinear transient stress analysis for the rotating crankshaft of a 6-cylinder inline engine was presented by Payer et al. (1995). A method was shown in their study which enabled the nonlinear transient analysis to be both highly sophisticated and
efficient for determining the fatigue behavior of crankshafts. They used a finite element program which uses two major steps to calculate the transient stress behavior of a rotating crankshaft, the structure of which is shown in Figure 2.14. The software automatically generates a solid element model of the crankshaft considering both flywheel and vibrational damper, see Figure 2.15. In order to create a beam mass model, shown in Figure 2.16, the strain energy method was used. Stiffness calculated with the solid element model of a crank throw was used to generate the beam mass model. This model was used for the calculation of the transient deformations of the crankshaft.

Prakash et al. (1998) modeled a complete crankshaft using the solid elements of ANSYS software. In order to verify the integrity of the FE model natural frequencies of the model was compared with those obtained from practical measurements. They evaluated the stress amplitudes using mode superposition method. As a prerequisite, reduced modal analysis was required for carrying out the mode superposition analysis. They selected the master degrees of freedom to exactly calculate the first few torsional natural frequencies. This method decouples the equations of motion and reduces computational effort required.

An analysis of the stress distribution inside a crankshaft crank was studied by Borges et al. (2002). The stress analysis was done to evaluate the overall structural efficiency of the crank, concerned with the homogeneity and magnitude of stresses as well as the amount and localization of stress concentration points. Due to memory limitations in the computers available, the crank model had to be simplified by mostly restricting it according to symmetry planes. In order to evaluate results from the finite element analysis a 3D photoelasticity test was conducted. The area of major interest was
that closest to crankpin bearing, usually critical to bending loads. An initial model was established as shown in Figure 2.17. Initial tentative meshing was performed over this 3D geometry, with several degrees of overall refinement and of local refinement. Due to limitations in computer memories, solving this initial model was not possible. According to experimental information from a photoelastic model, the theoretical model could be geometrically simplified by restricting symmetry planes. This results in the geometrically restricted symmetrical model shown in Figure 2.17. For the final model, Figure 2.18 shows (1) the applied load, (2) displacement restrictions which are the cutting planes for crank isolation, and (3) the symmetry plane.

Chien et al. (2005) studied the influence of the residual stress induced by the fillet rolling process on the fatigue process of a ductile cast iron crankshaft. The stress concentration near the fillet area of the crankshaft section was investigated by a 2D elastic finite element analysis in the ABAQUS software. Then, an elastic-plastic 2D plane strain FEA was conducted to obtain the residual stress distributions near the fillet due to the rolling process. The portion of the crankshaft section and the rollers were modeled by eight-node plane strain quadrilateral continuum elements with a reduced integration scheme, shown in Figure 2.19. A relatively fine mesh near the fillet was generated in order to accurately capture the characteristics of the stress field in this stress concentration area. A magnified finite element mesh of the crankshaft section near the fillet and a portion of the primary roller are shown in Figure 2.20.
2.5 Optimization of Crankshafts with Geometry, Material, Manufacturing, and Cost Considerations

Crankshaft is among large volume production components in the internal combustion engine industry. Weight and cost reduction of this component will result in high cost savings. Weight reduction of a crankshaft will also increase the fuel efficiency of the engine.

Geometry optimization

Development of the DCI crankshaft for the Nissan 60° V-6 engine was studied by Uchida and Hara (1984). It was aimed to reduce the web thickness while maintaining the performance of the crankshaft used before. This resulted in shortening the engine length. They used the finite element method to perform structural analyses. The analyses was necessary to set the absolute minimum dimensions for the cylinder pitch as well as each of the parts. The objective of their study was to reduce available length for various engine installation types, reduction of friction for better fuel economy, and good stiffness for lower noise.

Henry et al. (1992) used the durability assessment tool developed by RENAULT to optimize the crank web design of a turbocharged diesel engine crankshaft. The geometry of the web design was changed according to reasonable geometries in order to reduce the maximum stress. This constraint allowed changes in the web width and pin side web profile. The initial and modified webs are shown in Figures 2.21 (a) and (b). An increase of 11% for the fatigue safety factor for fillet P2 and 19% for fillet P1 (see Figure 2.12) resulted from the crank web modifications. In the fatigue improvement, two mechanisms contribute to the stress reduction, the first is the geometric reinforcement of
the webs, and the second is the decrease in maximum main bearing reaction on the fifth journal due to the influenced change in crank balancing.

Powertrain Engineering Tool (PET), developed at Ford Powertrain and Vehicle Research Laboratory by Mikulec et al. (1998) is a powertrain computer program that allows rapid development of preliminary powertrain component geometry based on the engine performance and friction. PET program uses its integrated design rules and a non-linear SQP-based (Sequential Quadratic Programming) geometry optimizer to calculate the powertrain component geometries. The simplified flow diagram of the PET component optimization scheme associated with the SQP is illustrated in Figure 2.22. They presented some initial designs and the optimized geometries given by the program in their study.

**Inducing residual stress by shot peening**

Shot peening is a carefully controlled process of blasting a large number of hardened spherical or nearly spherical particles (shot) against the softer surface of a part. Each impingement of a shot makes a small indentation in the surface of the part, thereby inducing compressive residual stresses, which are usually intended to resist fatigue fracture or stress-corrosion cracking.

Burrell (1985) studied the effect of controlled shot peening on the fatigue properties of different automotive components, including crankshafts. Six different crankshafts from different engines were investigated. The critically stressed location of each crankshaft was the journal bearing fillet. The high stress point was the bottom side of the fillet when the journal is in the top dead center position during the firing cycle. Each fatigue test was performed on single journal bearing of crankshafts. A high
frequency bending load was applied for the fatigue test and the load was monitored during the test by strain gages mounted in the fillets. The following results were obtained for the crankshafts mentioned:

- 25% increase in fatigue strength for a 6 cylinder diesel engine forged steel (AISI 15B55) crankshaft
- 40% increase in fatigue strength for a 6 cylinder diesel engine forged steel (AISI 4340H) crankshaft
- 31% increase in fatigue strength for a 6 cylinder diesel engine forged steel (AISI 1046) crankshaft
- 12.5% increase in fatigue strength for a 6 cylinder diesel engine nodular cast iron crankshaft
- 48% increase in fatigue strength for a 2 cylinder 4 cycle gasoline engine cast malleable iron (ARMASTEEL 88-M) crankshaft
- 17.2% increase in fatigue strength for a 16 cylinder diesel engine forged steel (C1046) crankshaft

The above data indicate that the fatigue strength of either forged steel or cast iron crankshafts increases as a result of shot controlled peening process, compared with the same unpeened component. Figure 2.23 shows the effect of controlled shot peening on a 6 cylinder diesel engine forged steel crankshaft.

**Inducing residual stress by fillet rolling**

Since fatigue fracture initiated near the fillets is one of the primary failure mechanisms of automotive crankshafts, fillet rolling process has been used to improve the fatigue lives of crankshafts for many years (Love et al., 1954). The fillet rolling
process induces compressive residual stresses near the fillet surface. The compressive residual stresses lower the fatigue driving stresses near the fillet surface due to operating loads and consequently increase the fatigue lives of crankshafts.

Kamimura (1985) performed a study on the effect of fillet rolling on fatigue strength of ductile cast iron crankshaft. He conducted bench tests of crankshaft pin samples and fatigue tests on test pieces in order to study the fatigue strengths of fillet rolled crankshafts and specimens. His study also showed that optimum deep rolling method could increase the bending fatigue strength by 83% over conventional ductile iron crankshafts not fillet rolled. Test results of his study are presented in Figure 2.24, which shows an S-N diagram for each of the non-deep rolled fillets and for eight different rolling forces.

Park et al. (2001) showed that without any dimensional modification, fatigue life of crankshaft could be improved significantly by applying various surface treatments. Fillet rolling and nitriding were the surface treatment processes that were studied in their research. Figure 2.25 shows relative S-N curve rig test results with various treatments; nitriding, fillet rolling (500 and 900 kgf), and crankshaft section bare samples. From the figure, it is seen that the standard bare sample has a fatigue limit of 10 kN. Fillet rolled specimens with 500 kgf load exhibit 14 kN fatigue limit, whereas with 900 kgf load more than 18 kN fatigue limit is obtained.

Above experimental data clearly indicate that fillet rolling can dramatically increase the fatigue performance of crankshafts. The fillet rolled at 900 kgf force shows more than 80% increase in fatigue limit, while the fillet rolled at 500 kgf force exhibits relatively lower improvement of 40% in fatigue limit. Therefore, higher rolling force
results in better fatigue strength as a result of inducing higher compressive stress on the fillet surface. However, there is a certain limit for applying residual stress and additional load above that level results in excess plastic deformation. Thus, it is important to decide the optimum level of rolling force, which was experimentally found to be 700~900 kgf rolling force for crankshaft section specimens in Park et al. study.

An extensive study was performed by Chien et al. (2004) about fatigue analysis of ductile cast iron (SAE J434C D5506) crankshaft sections under bending with consideration of the residual stresses induced by the fillet rolling process. They used a two-dimensional elastic finite element model to investigate the stress concentration near the fillet of the crankshaft section under bending without consideration of residual stress. A two-dimensional elastic-plastic FEA was created in order to study the plastic zone development and the residual stress distribution near the crankshaft fillet induced by the fillet rolling process. After the rolling process, a bending moment was applied to the crankshaft section. The fatigue failure of the crankshaft near the fillet area under bending moment was investigated considering the effect of the residual stress induced by the fillet rolling process based on a linear elastic fracture mechanics approach. Finally, the results of the fillet rolling FEA simulation were verified by the fillet surface profiles measured by the shadowgraphs taken before and after the fillet rolling process. The fatigue life of the component was studied using the resonant bending fatigue test on crankshaft sections, as shown in Figure 2.26. Figure 2.27 shows the radial distributions of the hoop stress, radial stress and shear stress of the investigated crankshaft at the critical angle ($\theta = 52.35^\circ$) on the fillet, based on the two dimensional plane strain model.
Nitriding the fillet area for improving fatigue limit

Nitriding is a surface hardening process that is applied only to certain type of steels. This process creates a finish that is the hardest surface attainable using heat treatment processes. The process consists of maintaining a work-piece in a 500 degree Celsius ammonia atmosphere for up to 100 hours. Under these conditions atomic nitrogen combines with surface iron to form iron nitride. The nitrogen slowly diffuses away from the surface, as long as the proper temperature is maintained. Therefore, the resulting case thickness depends on length of heat treatment.

Park et al. (2001) studied fatigue behavior of crankshafts investigating effect of nitriding surface finish on the fillet area of the crankshafts. From Figure 2.25 it can be seen that the standard bare crankshaft section had a fatigue limit of 10 kN and the lower hardness crankshaft section exhibited a fatigue limit of 9 kN. In the case of nitriding these crankshaft sections, the lower hardness crankshafts had a fatigue limit of 16 kN and the standard bare crankshaft sections fatigue limit increased to 18 kN, which is about 60% improvement. Therefore, it was suggested that the best combination of crankshaft materials is to apply proper surface modification to inexpensive steels with lower hardness to compromise both the cost and quality. In this study, the optimum level of nitriding depth was suggested to be at least 400 µm.

TR\textsuperscript{1} method in forging crankshafts

TR forging method is an advanced forging process to shape metallic components such as a crankshaft from a commercially available hollow pipe. The hollow pipe, which has a predetermined wall thickness, is squeezed into a hollow blank pipe having a predetermined shape and wall thickness. A region of the hollow blank pipe is then heated,

\textsuperscript{1} Initials of Prof. Tadeusz Rut, the author of the TR method of forging crankshafts.
and the heated region is upset and bent into a crankshaft, for example. The hollow blank pipe is upset and bent by dies positioned one on each side of the gripped portion of the blank and moving toward each other, and also by punches. After the crankshaft is forged, one of the punches is released from the crankshaft before the dies are separated therefrom (Ando et al., 1992).

The TR forging method has the following advantages when manufacturing crankshafts:

- material saving
- reduced labor cost
- high quality forging (e.g. proper grain flow)
- possibility of forging bigger crankshafts on relatively smaller forging presses

Figure 2.28 shows vertical cross-sectional view of a forging machine and the hollow blank pipe which starts being formed into a crankshaft. Figure 2.29 shows cross-sectional views of the TR forging sequences where a pipe is successively squeezed into a hollow blank pipe and forged into a crankshaft.

**Material and cost optimization**

An extensive study was performed by Nallicheri et al. (1991) on material alternatives for the automotive crankshaft based on manufacturing economics. They considered steel forging, nodular cast iron, micro-alloy forging, and austempered ductile iron casting as manufacturing options to evaluate the cost effectiveness of using these alternatives for crankshafts. Technical cost modeling method was used in their study to estimate the manufacturing costs of various material alternatives. They concluded that the production volume of the crankshaft and the requirements of the engine are predominant
factors in cost effective production route for this application. The cast iron crankshaft offered the most cost effective manufacturing process, but the properties offered by this production method have to be sufficient for the engine design. If the design requires better mechanical properties, then other alternatives must be considered. The selection of the best alternative depends upon the production volume in a year. At production volume above 200,000 parts/year, microalloyed steel forgings offered the most cost effective high performance crankshaft. They attributed this to the die and machining tool lives, which are improved for the microalloyed steel forging, but at lower than 200,000 parts/year production volume the cost savings obtained do not compensate for higher raw material cost of the microalloyed forged steel. ADI crankshafts were cost effective at low production runs (below 180,000 parts/year). Sensitivity with respect to volume production was evaluated as shown in Figure 2.30. Based on their assumptions, raw material was about 30% of the final cost, whereas, machining cost was 47% for the forged steel crankshaft. The raw material cost of the microalloyed steel forging crankshaft was 38%, which is higher than that of forged steel, but the machining percentage cost was 43% which reduced the cost of the final component. In addition, the heat treatment process was eliminated for the microalloyed steel crankshaft, which was 3.5% of the final cost of the forged steel, resulting in further final cost reduction.

A study was performed to examine the cost reduction opportunities to offset the penalties associated with forged steel, with raw material and machinability being the primary factors evaluated by Hoffmann and Turonek (1992). Materials evaluated in their study included medium carbon steel SAE 1050 (CS), and medium carbon alloy steel SAE 4140 (AS); these same grades at a sulfur level of 0.10%, (CS-HS and AS-HS); and two
micro-alloy grades (MA1 and MA2). The material selection was based on fatigue strength requirements and potential cost benefits. The micro-alloy grades evaluated offered cost reduction opportunities over the original design materials. The micro-alloy grade could reduce the finished cost by 11% to 19% compared to a quenched and tempered alloy steel (SAE 4140), and by 7% to 11% compared to a quenched and tempered carbon steel (SAE 1050). In addition, the micro-alloy grades met or exceeded the fatigue strength of the original materials for the applications studied, and had better machinability characteristics.
Figure 2.1  Crankshaft workpiece and lower forging die for a four cylinder crankshaft (Shamasundar et al. 2003).

Figure 2.2  Four cylinder forged crankshaft (Shamasundar et al. 2003).

Figure 2.3  Incomplete four cylinder forged crankshaft (Shamasundar et al. 2003).
Figure 2.4  a) Detail of a journal of a crankshaft, and b) Detail of journal of another crankshaft in the study by (Silvia, 2003).

Figure 2.5  A V-6 sample failed with a 2 Hz resonance shift with a 9.5 mm crack (Spiteri et al., 2005).
Figure 2.6 Photograph of test piece 3R after fracture at fillet A. Fatigue crack extending 2.2 cm along the edge fillet (Baxter, 1993).

Figure 2.7 V-8 crankshaft critical sections (Jenson, 1970).
Figure 2.8  Strain gage bridge placement on crankshaft critical sections (Jenson, 1970).

Figure 2.9  Typical trace of crankshaft loads (Jensen, 1970).
Figure 2.10 Finite element mesh for a crankshaft-flywheel assembly (Mourelatos, 1995).

Figure 2.11 A single throw model of crankshaft for finite element analysis (Uchida and Hara, 1984).
Figure 2.12  A finite element crankshaft model (Henry et al., 1992).

Figure 2.13  Stress prediction of crankshaft using a neural network structure (Shiomi and Watanabe, 1995).
Figure 2.14  Flow-chart for transient stress analysis of crankshaft (Payer et al., 1995).
Figure 2.15  Solid element model of a crankshaft (Payer et al., 1995).

Figure 2.16  Mass model of a crankshaft (Payer et al., 1995).
Figure 2.17  Complete geometry of a single throw of a crankshaft (initial model), (Borges et al., 2002).

Figure 2.18  Geometrically restricted, symmetrical model (final model) of a crankshaft section, with loading (1), restraints (2), and symmetry plane (3), in the study of (Borges et al., 2002).
Figure 2.19  A two dimensional finite element model of a portion of crankshaft section, with a primary roller and a secondary roller in the study by (Chien et al., 2004).

Figure 2.20  A magnified finite element mesh of the crankshaft section near the fillet and a portion of the primary roller in the study by (Chien et al., 2004).
Figure 2.21 (a) Initial crank web design, and (b) Modified crank web design in the study by (Henry et al., 1992).

Figure 2.22 Simplified flow chart of PET geometry optimization by (Mikulec et al., 1998).
Figure 2.23  Effect of controlled shot peening on the fillet area of a 6 cylinder diesel engine crankshaft made of AISI 4340H Forged steel (Burell, 1985).

Figure 2.24  S-N diagram of deep rolled ductile cast iron crankshaft with different rolling forces (Kamimura, 1985).
Figure 2.25  S-N curve rig test results with various surface treatments from the study by (Park et al., 2001).

<table>
<thead>
<tr>
<th>No.</th>
<th>Specimens</th>
<th>Fatigue limit (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Fillet rolling (900kgf)</td>
<td>21</td>
</tr>
<tr>
<td>2</td>
<td>Nitriding</td>
<td>18†</td>
</tr>
<tr>
<td>3</td>
<td>Nitriding (Low hardness)</td>
<td>16</td>
</tr>
<tr>
<td>4</td>
<td>Fillet rolling (500kgf)</td>
<td>14</td>
</tr>
<tr>
<td>5</td>
<td>Bare sample</td>
<td>10</td>
</tr>
<tr>
<td>6</td>
<td>Bare sample (Low hardness)</td>
<td>9</td>
</tr>
</tbody>
</table>

Figure 2.26  A crankshaft section used in the resonant bending fatigue test in the study by (Chien et al., 2004).
Figure 2.27  Radial distributions of the hoop stress, radial stress and shear stress of the crankshaft at $\theta = 52.35^\circ$ based on two-dimensional plane strain model (a) under bending, (b) roller down, (c) roller released and (d) summation of (a) and (c). (Chien et al., 2004).
Figure 2.28 Vertical cross-sectional view of an apparatus for forming a shaped metallic article, the view showing the hollow blank pipe which starts being formed into a crankshaft (Ando et al., 1992).
Figure 2.29    Fragmentary longitudinal cross-sectional views showing the manner in which a pipe is successively squeezed into a hollow blank pipe and forged into a crankshaft according to a TR forging process (Ando et al., 1992).
Figure 2.30  Finished piece costs; alternative processes as a function of annual production volume (Nallicheri et al., 1991).
3 Dynamic Load Analysis of the Crankshaft

The crankshaft experiences a complex loading due to the motion of the connecting rod, which transforms two sources of loading to the crankshaft. The main objective of this study was the optimization of the forged steel crankshaft which requires accurate magnitude of the loading on this component that consists of bending and torsion. The significance of torsion during a cycle and its maximum compared to the total magnitude of loading should be investigated to see if it is essential to consider torsion during loading or not. In addition, there was a need for obtaining the stress variation during a loading cycle and this requires FEA over the entire engine cycle.

The main objective of this chapter is to determine the magnitude and direction of the loads that act on the bearing between connecting rod and crankshaft, which was then used in the FEA over an entire cycle. An analytical approach was used on the basis of a single degree of freedom slider crank mechanism. MATLAB programming was used to solve the resulting equations.

The analytical approach was solved for a general slider crank mechanism which results in equations that could be used for any crank radius, connecting rod geometry, connecting rod mass, connecting rod inertia, engine speed, engine acceleration, piston diameter, piston and pin mass, pressure inside cylinder diagram, and any other variables of the engine. This analytical approach also helped to verify that the inputs in the ADAMS View software were correct. However, since changing variables in the
analytical approach using MATLAB programming code was more convenient, the results of ADAMS View software were used as verification of the analytical solutions.

In summary, this chapter explains the analytical approach steps and the equations that could be used in MATLAB to obtain angular velocity and acceleration of connecting rod, linear velocity and acceleration of piston assembly, and the most important forces between different joints in the mechanism. It is shown that the results from the analytical approach were verified by a simple model in ADAMS. How the output from the analytical approach is used in FEA is also discussed. It should be pointed out that in this analysis it was assumed that the crankshaft rotates at a constant angular velocity, which means the angular acceleration was not included in the analysis. However, in a comparison of forces with or without considering acceleration, the difference was found to be less than 1%.

### 3.1 Analytical Vector Approach

The analytical approach is discussed in detail in this section. The slider-crank mechanism with a single degree of freedom considered for solving the equations of motion is as shown in Figure 3.1. The following procedure was performed to obtain different dynamic properties of moving components.

The angle $\theta$ shown in Figure 3.1 represents the crankshaft angle, which is used as the generalized degree of freedom in the mechanism; therefore every other dynamic property in this mechanism would be a function of this angle. Calculation of other dynamic properties of the mechanism such as angular velocity, angular acceleration, and forces at pin joints are shown in Appendix A.
The equations where used in MATLAB and provided the values of angular velocity and angular acceleration of the connecting rod, linear acceleration of center of gravity of the connecting rod, and forces at the connecting rod-piston bearing and connecting rod-crankshaft bearing. The advantage of using MATLAB programming is that any changes in the input could be made very easily and solution quickly obtained, whereas using commercial programs such as ADAMS requires much more time editing the input data. This advantage comes into consideration when optimization is to be performed on a component, since during optimization mass and/or some dimensions change and making these changes in the commercial software is time consuming. The complete MATLAB program used in this analysis is given in Appendix B.

3.2 Verification of Analytical Approach

The analytical approach used in this study was verified by 3D dynamic simulation of the crankshaft, connecting rod, and piston assembly. The analysis was based on simulation of the simple slider-crank mechanism which is shown in Figure 3.1. As can be seen in the figure, link AB is the crankshaft radius, link BC is the connecting rod length, and the slider is the piston assembly. For the purpose of this simulation crankshaft and connecting rod were digitized and the generated geometries were used to obtain the accurate location of the center of gravity of the connecting rod and the magnitude of its inertia. Since the only concerning factor in the piston assembly that would affect the dynamic of the mechanism is the mass, there was no need to generate the piston assembly geometry. Material density of 2840 kg/m³ (2.84E-6 kg/mm³) was used for the connecting rod, which is the density of the aluminum alloy used in the component. The crankshaft
AB rotational speed was taken to be the maximum operating speed of the engine, which is 3600 rev/min. It is shown in next Sections that the engine speed effect within the operating engine speed of 2000 rpm to 3600 rpm, is less than 15% on the force range applied to the crankshaft. This results in about 10% change in the stress range of the critically stressed location, which is discussed in the next Chapter. The 3D model of slider crank mechanism used in ADAMS is shown in Figure 3.2.

Details of slider crank mechanism used in ADAMS are tabulated in Table 3.1. These details and any other information required were input in the MATLAB program. The loading on the piston which is the slider in the slider crank mechanism was taken to be zero in this part of the analysis since the purpose of this analysis was just compare the results from MATLAB programming and ADAMS software.

Although the simulation was performed in a 3D space, the mechanism is a simple 2D linkage; therefore forces are expected to be in the plane of crankshaft motion. Therefore, forces in the longitudinal direction of the crankshaft would be zero in this case. Since the joints at different locations of this mechanism are pin joints, there would be no moment resistance.

The results of the analytical approach for the slider crank mechanism using the MATLAB program are plotted in Figures 3.3 through 3.6. The results from ADAMS software are also included in these figures for comparison. Figure 3.3 shows the variation of angular velocity of link BC over one complete cycle of the engine, which is 720 degrees or two cycles of crank rotation. The two curves coincide perfectly indicating agreement of the results from MATLAB programming with results from ADAMS software. Similarly, Figure 3.4 shows the variation of angular acceleration over an entire
cycle, Figure 3.5 shows the variation of forces at joint B defined in the global/non-rotating coordinate system at the engine speed of 2800 rev/min, which is the mean operating speed of the engine, and Figure 3.6 shows the same forces at joint B projected in the local/rotating coordinate system. As could be seen in the figures, it can be concluded that the results from MATLAB programming are accurate and reliable.

3.3 Dynamic Analysis for the Actual Crankshaft

The engine configuration from which the crankshaft was taken is shown in Table 3.2. The pressure versus crank angle of this specific engine was not available, so the pressure versus volume (thermodynamic engine cycle) diagram of a similar engine was considered. This diagram was scaled between the minimum and maximum of pressure and volume of the engine. The four link mechanism was then solved by MATLAB programming to obtain the volume of the cylinder as a function of the crank angle.

Figure 3.7 shows the scaled graph of pressure versus volume for this specific engine, and Figure 3.8 shows pressure versus crankshaft angle, which was used as the applied force on the piston during the dynamic analysis. It should be noted that the pressure versus volume of the cylinder graph changes as a function of engine speed, but the changes are not significant and the maximum pressure which is the critical loading situation does not change. Therefore, the same diagram was used for different engine speeds in this study. The results of the MATLAB programming are linear velocity and acceleration of the piston assembly, angular velocity and angular acceleration of the connecting rod, linear acceleration of center of gravity of the connecting rod, and forces
that are being applied to the bearing between the crankshaft and the connecting rod. The program was run for different engine speeds in the operating engine speed range.

Results from the MATLAB programming at the engine speed of 3600 rpm are plotted in Figures 3.9 though 3.14. Figures 3.9 and 3.10 show the variation of linear velocity and linear acceleration of the piston assembly over 720 degrees, respectively. Figures 3.11 and 3.12 show the angular velocity and angular acceleration of the connecting rod during a cycle. Note that variations of velocity and acceleration in both piston assembly and connecting rod from 0° to 360° are identical to their variation from 360° to 720°.

Figure 3.13 shows the variation of the force at the journal bearing between crankshaft and connecting rod defined in the global/non-rotating coordinate system. Figure 3.14 shows the variation of the same force defined in the local/rotating coordinate system. $F_x$ in Figure 3.14 is the force that causes bending during service life and $F_y$ is the force that causes torsion on the crankshaft. As can be seen in this figure, the maximum loading happens at the angle of 355° where the combustion takes place. The only difference between these figures is their reference coordinate system, therefore the magnitude, which is not dependent on the coordinate system chosen, is the same in both plots.

As the dynamic loading on the component is a function of engine speed, the same analysis was performed for different engine speeds which were in the range of operating speed for this engine (i.e. 2000 rpm which is the minimum engine speed and 2800 rpm). The variation of forces defined in the local coordinate system at 2000 rpm and 2800 rpm engine speeds are shown in Figures 3.15 and 3.16, respectively. Figure 3.17 compares the
magnitude of maximum torsional load and bending load at different engine speeds. Note from this figures that as the engine speed increases the maximum bending load decreases. The reason for this situation could be explained as follows. As mentioned previously, there are two load sources in the engine; combustion and inertia. It was pointed out that the maximum pressure in the cylinder does not change as the engine speed changes, therefore the load applied to the crankshaft at the moment of maximum pressure due to combustion does not change. This is a bending load since it passes through the center of the crank radius. On the other hand, the load caused by inertia varies as a function of engine speed. As the engine speed increases this force increases too. The load produced by combustion is greater than the load caused by inertia and is in the opposite direction, which means the sum of these two forces results in the bending force at the time of combustion. So as the engine speed increases the magnitude of the inertia force increases and this amount is deducted from the greater force which is caused by combustion, resulting in a decrease in total load magnitude. However, factors such as wear and lubrication are important at higher engine speeds. Further discussion of these issues is avoided since they are not of concern in this study and fatigue failure is the main focus.

In this specific engine with its dynamic loading, it is shown in the next chapter that torsional load has no effect on the range of von Mises stress at the critical location. The main reason for torsional load not having much effect on the stress range is that the maximums of bending and torsional loading happen at different times during the engine cycle. In addition, when the main peak of the bending takes place the magnitude of torsional load is zero.
The dynamic analysis of this single cylinder crankshaft is very similar to an automotive crankshaft which consists of several cylinders. The only difference is the number of applied loads to the mechanism which could be projected to the rotating plane of the crankshaft. In a multi-cylinder crankshaft the effect of combustion of other cylinders on one cylinder results in high torsional load which must be included in the analysis. Since the studied crankshaft belonged to a single cylinder engine, there would be no such effect. Therefore, the analysis could be performed without considering torsional load. The noise and vibration analysis of single cylinder and multi-cylinder crankshafts are similar. The longitudinal and radial displacements of a single throw, which consists of two main bearings, two crank webs, and a crankpin, under service load is measured in order to define the noise and vibration level of the crankshaft. Therefore, the analysis followed in this study could be implemented in the analysis of a single throw of a multi-cylinder crankshaft as well (http://www.steel.org/, 2007).

3.4 FEA with Dynamic Loads

There are two different approaches for applying the loads on the crankshaft to obtain the stress time history. One method is to run the FE model many times during the engine cycle or at selected times over 720° by applying the magnitude of the load with its direction in a way that the loading could define the stress-time history of the component. Another approach to obtain stresses at different locations at different times during a cycle is by superposition of the basic loading conditions. This involves applying a unit load in the basic conditions and then scaling the stresses from each unit load according to the dynamic loading. Then similar stress components are added together. In this study both
approaches were used for the engine speed of 3600 rpm to verify that results from both approaches are the same. After verification of results, the superposition approach was used by developing a code in Excel spread sheet to perform the necessary calculation and obtain the results for the stresses at different crankshaft angles.

Justification for selecting some locations over 720° was according to peaks and valleys of load variation. Three different graphs were used for selecting proper points to cover the entire cycle; bending, torsion, and total load magnitude. Figure 3.18 shows the variation of these loads versus crank angle. Selected points are labeled in this figure and the reason of selecting each point is indicated as follows:

a. Beginning of the cycle
b. A peak of the bending load
c. Valley of the bending load
d. Bending load of zero
e. Peak of torsional load
f. Peaks of bending load and total load magnitude
g. A valley of torsional load
h. Bending load of zero
i. A valley of total load magnitude
j. A valley of bending load
k. Selected to have smooth connectivity between the points before and after
l. A valley of total load magnitude and a peak of bending load
The same locations could be identified in the load history diagram of the crankshaft at other engine speeds. However, only engine speed of 2000 rev/min was selected for verification of results.

FE analysis was performed on these 12 points over 720 degrees, and the results where compared with the results from the superposition. The comparison showed errors less than 3% on each stress component, which verified the use of superposition in all other cases.

As the dynamic loading condition was analyzed, only two main loading conditions were applied to the surface of the crankpin bearing. These two loads are perpendicular to each other and their directions are shown in Figure 3.19 as $F_x$ and $F_y$. Since the contact surface between connecting rod and crankpin bearing does not carry tension, $F_x$ and $F_y$ can also act in the opposite direction to those shown in Figure 3.19. Any loading condition during the service life of the crankshaft can be obtained by scaling and combining the magnitude and direction of these two loads.
Table 3.1  Details of slider crank mechanism used in ADAMS software.

<table>
<thead>
<tr>
<th></th>
<th>Crank AB</th>
<th>Connecting rod BC</th>
<th>Slider C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calculated Mass (kg)</td>
<td>3.7191</td>
<td>283.35E-3</td>
<td>417.63E-3</td>
</tr>
<tr>
<td>$I_{xx}$ (kg-mm$^2$)</td>
<td>-</td>
<td>608.5844</td>
<td>-</td>
</tr>
<tr>
<td>$I_{yy}$ (kg-mm$^2$)</td>
<td>-</td>
<td>80.3227</td>
<td>-</td>
</tr>
<tr>
<td>$I_{zz}$ (kg-mm$^2$)</td>
<td>-</td>
<td>662.5235</td>
<td>-</td>
</tr>
<tr>
<td>$I_{xy}$ (kg-mm$^2$)</td>
<td>-</td>
<td>8.0467</td>
<td>-</td>
</tr>
<tr>
<td>$I_{xz}$ (kg-mm$^2$)</td>
<td>-</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>$I_{yz}$ (kg-mm$^2$)</td>
<td>-</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>Length (mm)</td>
<td>36.985</td>
<td>120.78</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 3.2  Configuration of the engine to which the crankshaft belongs.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Crankshaft radius</td>
<td>37 mm</td>
</tr>
<tr>
<td>Piston diameter</td>
<td>89 mm</td>
</tr>
<tr>
<td>Mass of the connecting rod</td>
<td>0.283 kg</td>
</tr>
<tr>
<td>Mass of the piston assembly</td>
<td>0.417 kg</td>
</tr>
<tr>
<td>Connecting rod length</td>
<td>120.78 mm</td>
</tr>
<tr>
<td>$I_{zz}$ of connecting rod about the center of gravity</td>
<td>$0.663\times10^{-3}$ kg-m$^2$</td>
</tr>
<tr>
<td>Distance of C.G. of connecting rod from crank end center</td>
<td>28.6 mm</td>
</tr>
<tr>
<td>Maximum gas pressure</td>
<td>35 Bar</td>
</tr>
</tbody>
</table>
Figure 3.1 Slider-crank mechanism.

Figure 3.2 Crankshaft, connecting rod, and piston assembly model in ADAMS/View software.
Figure 3.3 Angular velocity of link BC for the slider-crank mechanism. A comparison of the results obtained by MATLAB programming and ADAMS/View software at 2800 rpm crank speed.

Figure 3.4 Angular acceleration of link BC for slider-crank mechanism. A comparison of the results obtained by MATLAB programming and ADAMS/View software at 2800 rpm crank speed.
Figure 3.5  Forces at joint B defined in the global/non-rotating coordinate system, for the slider-crank mechanism. A comparison of results obtained by MATLAB programming and ADAMS/View software at 2800 rpm crank speed.

Figure 3.6  Forces at joint B defined in the local/rotating coordinate system, for the slider-crank mechanism. A comparison of results obtained by MATLAB programming and ADAMS/View software at 2800 rpm crank speed.
Figure 3.7  Pressure versus volume of the engine used to calculate the pressure versus crankshaft angle diagram (Taylor, 1985).

Figure 3.8  Pressure versus crankshaft angle used to calculate the forces at the surface of the piston.
Figure 3.9  Variation of linear velocity of the piston assembly over one complete engine cycle at crankshaft speed of 3600 rpm.

Figure 3.10  Variation of linear acceleration of the piston assembly over one complete engine cycle at crankshaft speed of 3600 rpm.
Figure 3.11  Variation of angular velocity of the connecting rod over one complete engine cycle at crankshaft speed of 3600 rpm.

Figure 3.12  Variation of angular acceleration of the connecting rod over one complete engine cycle at crankshaft speed of 3600 rpm.
Figure 3.13  Variation of the force components over one complete cycle at the crank end of the connecting rod defined in the global/non-rotating coordinate system at crankshaft speed of 3600 rpm.

Figure 3.14  Variation of the force components over one complete cycle at the crank end of the connecting rod defined in the local/rotating coordinate system at crankshaft speed of 3600 rpm.
Figure 3.15  Variation of the force components over one complete cycle at the crank end of the connecting rod defined in the local/rotating coordinate system at crankshaft speed of 2000 rpm.

Figure 3.16  Variation of the force components over one complete cycle at the crank end of the connecting rod defined in the local/rotating coordinate system at crankshaft speed of 2800 rpm.
Figure 3.17  Comparison of maximum, minimum, mean, and range of load between bending and torsional load at different engine speeds.

Figure 3.18  Variation of the force components over one complete cycle at the crank end of the connecting rod defined in the local/rotating coordinate system at crankshaft speed of 2000 rpm, with labels on critical locations on each graph.
Figure 3.19  Crankshaft geometry and bending ($F_x$), torsional ($F_y$), and longitudinal ($F_z$) force directions.
4 Stress Analysis and FEA

This chapter discusses geometry generation used for finite element analysis, describes the accuracy of the model and explains the simplifications that were made to obtain an efficient FE model. Mesh generation and its convergence are discussed. Using proper boundary conditions and type of loading are important since they strongly affect the results of the finite element analysis. Identifying appropriate boundary conditions and loading situation are also discussed. Finite element models of two components were analyzed; the cast iron crankshaft and the forged steel crankshaft. Since these two crankshafts are from similar engines, the same boundary conditions and loading were used for both. This facilitates proper comparison of this component made from two different manufacturing processes. The results of finite element analysis from these two crankshafts are discussed in this chapter. Above mentioned FE models were used for dynamic analysis considering the boundary conditions according to the mounting of the crankshafts in the engine.

In order to evaluate the FEA results, a component test was conducted with strain gages. FEA boundary conditions were changed according to the test setup. Strain gages were mounted on the forged steel crankshaft and results from FE analysis and experimental data were compared in order to show the accuracy of the FE model.
Finally, results from dynamic FE analysis, which consist of stress history at different locations, were used as the input to the optimization process discussed in Chapter 5.

4.1 Finite Element Modeling

Finite element modeling of any solid component consists of geometry generation, applying material properties, meshing the component, defining the boundary constraints, and applying the proper load type. These steps will lead to the stresses and displacements in the component. In this study, similar analysis procedures were performed for both forged steel and cast iron crankshafts.

4.1.1 Generation of the Geometry of Crankshafts

The dimensions of the crankshafts were measured using a clipper and a CMM (coordinate measuring machine) with the accuracy of 0.025 mm (0.001 in) and 0.0025 mm (0.0001 in), respectively. Having accurate dimensions of both crankshafts solid models were generated using I-DEAS Master Modeler.

The solid model generated for the cast crankshaft is shown in Figure 4.1 and a picture of the cast crankshaft from which the geometry was generated is shown in Figure 4.2. As can be seen in the picture of the cast crankshaft the gear on the rear side of the main bearing is not included in the digitized model, instead the plane next to it is extruded to cover the section which the gear covers. This simplification is reasonable since the gear is press fit at this location and the gear tooth does not have any effect on stresses at fillet areas where high stress gradient exists. It should be mentioned that
simplification on the gear will not affect the stiffness of the model because the boundary condition used on this side of the crankshaft is a sliding edge and moving its location along the main bearing shaft will not change the stress results. In addition, the significant effect of bending load was observed to be in the cross section of the crankpin bearing and crank web. Therefore, simplifications on the main bearings will not affect stresses at critical areas. Another simplification done on the cast iron crankshaft was neglecting the cap at the end of oil way on the bearing of the connecting rod. This cap is pressed in its place to seal the drilled hole to prevent oil to flow out. The absence of this cap does not affect stresses at any location; therefore the cap was not included in the model. The last simplification done on the cast iron crankshaft was neglecting the slight slope on the crank web from outside toward centerline of main bearings. The slope is a result of manufacturing process. The molds in casting are such that the molded part could easily slip out the molds; as a result, walls that are perpendicular to the mold movement must have a positive slope. Since the change in the web thickness was small, by averaging thickness over the entire crank web, the crank web was modeled with uniform thickness. This simplification is acceptable since stresses at this location are very low and the thickness does not change the results at the fillet areas, which are critical locations.

Figure 4.3 shows the solid model of the forged steel crankshaft, and Figure 4.4 shows a picture from the same view of the received crankshaft. As can be seen the threaded front shaft is not threaded in the model, since this part is out of loading and boundary conditions and has no effect on stresses at different locations. Another simplification made to forged steel crankshaft model was in the crank web slope, similar to the cast iron crankshaft. In the manufacturing process of forging there is a need of the
same slight slope as in casting process on the component in order to ease removing the forged component from the forging die. The web slope in the forged component is less than the web slope in the cast iron crankshaft. Again, an average value was used as the uniform thickness on the crank web in this model.

Drilled holes on the counter weights in order to balance the crankshafts and inside threads on holes at the back of the crankshafts were not included in the models since their presence makes the geometry complicated but they do not affect stresses at critical locations.

The cast iron crankshaft weight as measured on a weighing scale was 3.58 kg, and the weight of the forged steel crankshaft was 3.80 kg. The difference between the generated models and the actual crankshafts were about 7% and 2%, for the cast iron and forged steel crankshafts, respectively. The reason for 7% difference in the cast crankshaft is not including the gear tooth in the solid model, which if considered, the difference will reduce to only 3%. Material properties used in both models are tabulated in Table 4.1.

Another important characteristic for this component is the dynamic balance of the geometry; this property could be verified by the location of center of gravity. Examining the location of center of gravity of both digitized models showed a very close distance from the main bearings center line. By adding the drilled holes on the counter weight of the actual crankshafts, the center of gravity will coincide with the main bearing center line, indicating the component would be in proper dynamic balance.

This weight comparison between the actual crankshafts and corresponding models and dynamic balance are indications of accuracy of the generated models. Figures 4.5 and
4.6 show detailed drawings of the forged steel and cast iron crankshafts, respectively, based on the measured dimensions.

4.1.2 Mesh Generation

FEA analysis was performed on both crankshafts for the dynamic load analysis, as well as for the test setup. Since boundary conditions of dynamic FEA and test setup FEA are different, separate FE models were needed. In this section, meshing of both dynamic FEA and test setup FEA are presented for the forged steel and cast iron crankshafts.

4.1.2.1 Dynamic FEA

Quadratic tetrahedral elements were used to mesh the crankshaft finite element geometry. Tetrahedral elements are the only option for meshing the imported complex geometries to the ABAQUS software. Using linear tetrahedral elements will result in a rigid model with less accuracy, whereas using quadratic tetrahedral elements will increase the accuracy and lessen the rigidity of the geometry. In order to mesh the geometry with this element type, the free meshing feature of ABAQUS software was used. In this feature, the global mesh size could be defined, while for critical locations free local meshing could be used to increase the number of elements for accurate stresses at locations with high stress gradients.

Convergence of stress at different locations was considered as the criterion for mesh size and number of elements selection. Figure 4.7 shows von Mises stress magnitude at six locations on fillet areas versus number of elements in the forged steel
crankshaft geometry for a load of 20.3 kN applied in the F_x direction shown in Figure 3.19. As could be seen from Figure 4.7, with the increase of element numbers, especially in the fillet areas, the stresses converge. Satisfactory results were obtained using 119,337 elements for the forged steel crankshaft and 121,138 elements for the cast iron crankshaft, corresponding to a global mesh size of 5.08 mm and a local mesh size of 0.762 mm in each model. This local mesh size results in having 5 elements in the radius of the fillet areas for both crankshafts.

In order to have more efficient models, different element sizes were used for the final models which increased the number of elements to 122,441 and 128,366 for the forged steel and the cast iron crankshafts, respectively. The selection of different sizes for elements was made to obtain a uniform growth of element size as the element size changed through the geometry. Figure 4.8 shows element sizes used at different locations for the forged steel crankshaft and Figure 4.9 shows the element sizes used for the cast iron crankshaft. Figures 4.10 and 4.11 show the meshed geometry for forged and cast components, respectively, which were meshed using the mentioned mesh size growth.

### 4.1.2.2 Test Assembly FEA

FEA was also conducted for the bench test assembly of the forged steel crankshaft. According to the fixture of the component test assembly a separate model was created. The only difference in the mesh is the size of mesh at the part where the crankshaft is gripped in the fixture. Figure 4.12 shows the local mesh size used in the test assembly FEA. The reason for using a fine mesh on the boundary of the gripped location is to measure stress around the oil hole which act as a stress concentration. Considering the above mentioned considerations, the crankshaft model resulted in 137,779 elements.
4.1.3 Loading and Boundary Conditions

4.1.3.1 Dynamic FEA

The engine manual of the forged steel crankshaft was used to determine the proper boundary conditions at bearing locations. Figure 4.13 was taken from the engine manual of the forged steel crankshaft and shows different components at a cut view of the engine. It can be seen that the crankshaft is constraint with a ball bearing (number 9 on Figure 4.13) from one side and with a journal bearing on the other side. The ball bearing is press fit to the crankshaft and does not allow the crankshaft to have any motion other than rotation about its main central axis. Since only 180 degrees of the bearing surfaces facing the load direction constraint the motion of the crankshaft, this constraint was defined as a fixed semicircular surface as wide as the ball bearing width on the crankshaft. The other side of the crankshaft is on a journal bearing. Therefore, this side was modeled as a semicircular edge facing the load at the bottom of the fillet radius fixed in a plane perpendicular to the central axis and free to move along central axis direction.

Figures 4.14 and 4.15 show these defined boundary conditions in the FE model of forged steel and cast iron crankshafts, respectively. Boundary conditions rotate with the direction of the load applied such that the inner face of the fixed semicircular surface and sliding ring face the direction of the load. For example, if the load is applied downward in Figure 4.14 (opposite direction of axis 2) the inner curve of both boundary conditions will face upward in the same figure (positive direction of axis 2).

Definition of a fixed edge is based on the degrees of freedom in a journal bearing, which allows the crankshaft to have displacement along its central axis. Also, the section
that is located on the journal bearing can have limited rotation in the direction perpendicular to the plane which central axis and load vector lay in (i.e. $F_x$ in Figure 3.19).

The distribution of load over the connecting rod bearing is uniform pressure on 120° of contact area, shown in Figures 4.14 and 4.15. This load distribution is based on experimental results from Webster et al. (1983). The explanation of load distribution in the Webster et al. study is for connecting rods, but since the crankshaft is in interaction with the connecting rod, the same loading distribution will be transmitted to the crankshaft. For pressure $P_0$ on the contact surface, the total resultant load is given by:

$$F = \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} P_0 \cos(\phi) r t \, d\phi = P_0 r t \sqrt{3}$$  \hspace{1cm} (4.1)

where $r$ is the crankpin radius and $t$ is the crankpin length. As a result, the pressure constant is given by:

$$P_0 = \frac{F}{r t \sqrt{3}}$$  \hspace{1cm} (4.2)

Force $F$, which is the magnitude of the total force applied to the crankshaft, can be obtained from dynamics analysis at different angles. According to the geometry of the forged steel crankshaft a unit load of 1 kN will result in the pressure of 1.142 MPa, as follows:

$$P_0 = \frac{1000}{18.48 \times 27.37 \times \sqrt{3}} = 1.142 \text{ MPa}$$

The same boundary conditions and loading were used for the cast iron crankshaft. Since some of the dimensions are different in the two crankshafts, the applied pressure resulting from a unit load of 1 kN is calculated to be 1.018 MPa,
\[ P_0 = \frac{1000}{16.51 \times 34.35 \times \sqrt{3}} = 1.018 \text{ MPa} \]

### 4.1.3.2 Test Assembly FEA

Figure 4.16 shows a schematic drawing of the fixture of test assembly. As can be seen one crankshaft end is gripped in the fixed column and the other end is gripped in a 44 cm arm to apply bending load. According to this assembly one side of the crankshaft which is in the column will be constrained in all degrees of freedom in the FE model. A load is then applied to the other side. Since the material behavior is fully elastic, stresses for any load magnitude could be obtained by scaling each stress component resulting from a unit load. These boundary conditions applied to the FE model of the test assembly are shown in Figure 4.17.

#### 4.2 Finite Element Analysis Results and Discussion

In Section 3.4 it was pointed out that the analysis conducted was based on superposition of four basic loadings in the FE analysis. The unit load applied on the connecting rod bearing was a pressure of magnitude 1.142 MPa and 1.018 MPa for forged steel and cast iron crankshafts, respectively. Note that the resultant load \( F \) was 1 kN and because of differences in dimensions of the two crankshafts, the pressure is somewhat different.

Section changes in the crankshaft geometry result in stress concentrations at intersections where different sections connect together. Although edges of these sections are filleted in order to decrease the stress level, these fillet areas are highly stresses
locations over the geometry of crankshaft. Therefore, stresses were traced over these locations.

Stress results from applying unit load on both crankshafts are tabulated in Table 4.2 for the forged steel crankshaft and Table 4.3 for the cast iron crankshaft. The stress components in Tables 4.2 and 4.3 were used to obtain stresses at different loading conditions by scaling these values by the magnitude of the applied load.

In order to obtain stresses at any location at different crank angles these tabulated data were used as explained below:

At a crank angle for which stress components are aimed to be calculated, load components at that crank angle defined in the local rotating coordinate system are taken from the dynamic analysis with consideration of their sign. Since the analysis is based on linear elastic behavior of the material, stress magnitude has linear relation with load, therefore stresses tabulated in Tables 4.2 and 4.3 are scaled according to the proper load component. Identical stress components are then added together resulting in stress components of the aimed loading situation. Replacing these stress components in the following equation gives the von Mises stress.

\[
\sigma_{\text{von Mises}} = \frac{1}{\sqrt{2}} \sqrt{(\sigma_{xx} - \sigma_{yy})^2 + (\sigma_{yy} - \sigma_{zz})^2 + (\sigma_{zz} - \sigma_{xx})^2 + 6(\sigma_{xy}^2 + \sigma_{xz}^2 + \sigma_{yz}^2)}
\]  

(4.3)

Following the above mentioned procedure, von Mises stress results for both FE models of forged steel and cast iron crankshafts were obtained.

**4.2.1 Finding the Critical Location**

FE analyses were performed on the forged steel crankshaft as well as the cast iron crankshaft at different engine speeds. Investigation of the FE models shows that the fillet
areas experience the highest stresses during service life of the crankshaft. Therefore, six points on the fillets were selected and labeled in Figure 4.18 for forged steel crankshaft and in Figure 4.19 for cast iron crankshaft.

Locations 1 and 6 are located on the boundary conditions of three of the loading conditions. For the loading condition, where the load direction is toward the center of the crankshaft, locations 1 and 6 are located far from boundary conditions (refer to Section 4.1.3.1 for boundary conditions). This loading condition is the only loading condition used at the time of maximum bending load, because at this time the torsional load is zero. Therefore, using the stress results and scaling them according to the maximum dynamic load at this moment will give the maximum stress at these locations.

Figures 4.20 and 4.21 show the von Mises stress with sign at these six locations at the engine speed of 2000 rpm for the forged steel and cast iron crankshafts, respectively. The sign of von Mises stress is determined by the sign of the principal stress that has the maximum absolute value. As can be seen from both figures, the maximum von Mises stress occurs at location 2, while other locations experience stresses lower than location 2. Therefore, other five locations were not considered to be critical in the rest of the analysis. Since locations 1 and 6 have lower stresses than location 2 at the critical loading condition, finding stresses at these locations for the other three boundary conditions was unnecessary. According to the obtained results, the maximum von Mises stress value at location 2 for the forged steel crankshaft is 186 MPa and 195 MPa for the cast iron crankshaft at the engine speed of 2000 rpm.

Since stress range and mean stress are the main controlling parameters for calculating fatigue life of the component, these parameters have to be calculated. Figures
4.22 and 4.23 show minimum, maximum, mean, and range of stress at selected locations on the forged steel and cast iron crankshafts, respectively, at the engine speed of 2000 rpm. As can be seen from these figures, location 2 has the highest maximum stress as well as the maximum value of stress range in both crankshafts. This location also has a positive mean stress, which has a detrimental effect on the fatigue life of the component. Therefore, location 2 is the critical location on both crankshafts, and any further discussion is with regards to this critical location on both crankshafts.

One of the main objectives of performing the dynamic FE modeling was to determine the design loads for optimization of the forged steel crankshaft. The maximum load that is applied to the crankshaft during its service life is the load corresponding to the peak gas pressure. Figure 3.8 indicated that the maximum gas pressure occurs at about 355° crank angle. Figures 4.20 and 4.21 also show that the maximum stress occurs at this crank angle. Therefore, this loading condition is the most severe case of loading resulting in the maximum magnitude of von Mises stress at location 2.

### 4.2.2 Finding Critical Engine Speed

As discussed in Section 3.3, dynamic load applied on the crankshaft is a function of engine speed. Figures 4.24 and 4.25 show comparison plots of von Mises stress at location 2 at different engine speeds for forged steel and cast iron crankshafts, respectively. As can be seen in these figures, with the increase of engine speed the maximum stress and, therefore, the stress range decreases, although not very significantly. Therefore, the critical engine speed will be the lowest operating engine speed, which is 2000 rpm according to the engine manual. This issue should not be
misunderstood as the higher the engine runs the longer the service life, since there are many other factors to consider in an engine. The most important issue when the engine speed increases is wear and lubrication. As these issues were not of concern in this study, further discussion is avoided. Engine speeds lower than 2000 rpm are transient, which means the engine speeds up in few seconds to its operating speed. Since an electric rotor starts the engine, combustion does not occur during the transient speed up and torque output is not taken from the engine. Therefore, speed engines lower than 2000 rpm are not considered.

4.2.3 Effect of Torsional Load

In this specific engine with its dynamic loading, it is shown that torsional load has no effect on the range of von Mises stress at the critical location. The main reason of torsional load not having any effect on the stress range is that the maxima of bending and torsional load happen at different times. In addition, when the peak of the bending load occurs the magnitude of torsional load is zero. Therefore, crankshafts are usually tested under bending fatigue load, as it was the case for the single cylinder crankshaft investigated in this study.

Stress magnitudes without considering torsion were calculated by substituting the value of zero for all \( F_y \) load components. Figures 4.26 and 4.27 show the von Mises stresses at location 2 at the engine speed of 2000 rpm considering torsion and without considering torsion for the forged steel and cast iron crankshafts, respectively. It can be seen that the stress-time history remains the same with and without considering torsional
load for both crankshafts. This is due to the location of the critical point which is not influenced by torsion since it is located on the crankpin bearing.

Other locations such as 1, 6, and 7 in Figures 4.18 and 4.19 experience the torsional load. Since locations 1 and 6 are located on the boundary conditions of the FE model, stress results at these locations are not reliable. Therefore, location 7 was selected as a point which is located far enough from the boundary conditions and also carries the effect of torsional load. Figures 4.28 and 4.29 show changes in minimum, maximum, mean, and range of von Mises stress at location 7 with considering torsion and without considering it at different engine speeds for the forged steel and cast iron crankshafts, respectively. The effect of torsion at the engine speed of 2000 rpm is about 16% increase in the stress range at this location for the forged steel crankshaft, and 18% increase for the cast iron crankshaft. It should be added that the effect of torsion decreases as the engine speed increases. Therefore, at higher engine speeds the effect of torsion is less than the mentioned percentages.

4.2.4 Validation of FEA Results with Experimental Results

Stress results obtained from the FE model of the forged steel crankshaft were verified by experimental component test. Strain gages were mounted at four locations in the midsection of the crankpin bearing of the forged steel crankshaft. These locations are labeled as a, b, c, and d in Figure 4.30. The reason for attaching strain gages at these locations is that the size of strain gages was not small enough to place them in the fillet areas. In addition, stress gradients in the fillet areas are high, therefore, values measured by strain gages at these locations would not be accurate.
Considering the test assembly boundary conditions in Section 4.1.3.2, the crankshaft experiences bending as a cantilever beam. Applying load in the direction of axis 2 in Figure 4.17 will result in stresses at locations a and b, and applying load in the direction of axis 1 in the same figure will result in stresses at locations c and d. Analytical calculations based on pure bending equation, $Mc/I$, show the magnitude of stresses at these four locations to be the same and equal to 72 MPa at these locations, for a 890 N load. The values obtained from experiments are tabulated in Table 4.4. FEA results are also shown and compared with experimental results in this table. As can be seen, differences between FEA and experimental results are less than 6.5% for different loading conditions. This is an indication of the accuracy of the FE model used in this study.

A comparison between the analytical calculations, experimental, and FEA results shows that for the complex geometry of the crankshaft, analytical approach does not result in accurate stresses, as shown in Table 4.4. Therefore, using FE models to calculate stresses were necessary.

### 4.2.5 Fatigue Life Calculations

Load variation over an engine cycle results in variation of stress. For proper calculations of fatigue damage in the component there is a need for a cycle counting method over the stress history. Using the rainflow cycle counting method (Stephens et al., 2000) on the critical location, stress-history plot (i.e. location 2 in Figure 4.18) shows that during an entire cycle only one peak is important and can cause fatigue damage in the component. The result of the rainflow count over the stress-time history of location 2 at
the engine speed of 2000 rpm is shown in Figures 4.31 and 4.32 for the forged steel and cast iron crankshafts, respectively. It is shown in these figures that in the stress history of the critical location only one cycle of loading is important and the other minor cycles have low stress amplitudes.

As a result of load analysis, the load $R$ ratio, which is the ratio of the minimum load to maximum load, applied to both crankshafts during service life is about $–0.30$ and $–0.16$ for the engine speed of 3600 rpm and 2000 rpm, respectively. Investigation of stress $R$ ratio, which is the ratio of minimum von Mises stress to maximum von Mises stress, according to the stress-time history at critical location 2 at the engine speed of 3600 rpm to 2000 rpm shows a variation of $–0.22$ to $–0.12$ for the forged steel crankshaft, and $–0.25$ to $–0.13$ for the cast iron crankshaft.
Table 4.1  Material properties for cast iron and forged steel (Williams et al. 2007).

<table>
<thead>
<tr>
<th>Material Property</th>
<th>Unit</th>
<th>Forged Steel</th>
<th>Cast Iron</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modulus of Elasticity</td>
<td>GPa</td>
<td>221</td>
<td>178</td>
</tr>
<tr>
<td></td>
<td>ksi</td>
<td>32,053</td>
<td>25,817</td>
</tr>
<tr>
<td>Poisson’s Ratio</td>
<td>-</td>
<td>0.30</td>
<td>0.30</td>
</tr>
<tr>
<td>Mass Density</td>
<td>kg/m$^3$</td>
<td>7833</td>
<td>7197</td>
</tr>
<tr>
<td></td>
<td>lb/in$^3$</td>
<td>0.283</td>
<td>0.26</td>
</tr>
</tbody>
</table>

Table 4.2  Stress components in MPa at locations labeled in Figure 4.18 on the forged steel crankshaft, resulting from unit load of 1 kN.

<table>
<thead>
<tr>
<th>Location Number</th>
<th>von Mises</th>
<th>S11</th>
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Table 4.3  Stress components in MPa at locations labeled in Figure 4.19 on the cast iron crankshaft, resulting from unit load of 1 kN.

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Table 4.4  Comparison of stress results from FEA and strain gages located at positions shown in Figure 4.30.

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* Difference between FEA results and experimental results
+ Difference between analytical results and experimental results
Figure 4.1   Generated geometry of the cast iron crankshaft.

Figure 4.2   Picture of the actual cast iron crankshaft from which the digitized model was generated.
Figure 4.3  Generated geometry of the forged steel crankshaft.

Figure 4.4  Picture of the actual forged steel crankshaft from which the digitized model was generated.
Figure 4.5 Drawing of the forged steel crankshaft with measured dimensions noted.
Figure 4.6  Drawing of the cast iron crankshaft with measured dimensions noted.
Figure 4.7  von Mises stress at locations 1 through 6 for forged steel crankshaft. Note that the convergence is achieved at all locations with 119,337 elements. This corresponds to a global element size of 5.08 mm and a local mesh size of 0.762 mm.
Figure 4.8  Element size at different locations on the forged steel crankshaft geometry. Note that the global mesh size is 5.08 mm. (a) back view (b) front view.
Figure 4.9  Element size at different locations on the cast iron crankshaft geometry. Note that the global mesh size is 5.08 mm. (a) back view (b) front view.
Figure 4.10  Meshed geometry of the forged steel crankshaft with 122,441 elements.

Figure 4.11  Meshed geometry of the cast iron crankshaft with 128,366 elements.
Figure 4.12  Element size at different locations on the forged steel crankshaft geometry for test assembly FEA. Note that the global mesh size is 5.08 mm.

Figure 4.13  Forged steel crankshaft in the engine block (taken from the engine manual).
Figure 4.14  Boundary conditions of dynamic model of the forged steel crankshaft with the load applied on the upper part of the connecting rod bearing.

Figure 4.15  Boundary conditions of dynamic model of the cast iron crankshaft with the load applied on the upper part of the connecting rod bearing.
Figure 4.16  Schematic of test assembly fixture for the forged steel crankshaft.

Figure 4.17  Boundary conditions of test assembly model of the forged steel crankshaft with the load applied as a concentrated force.
Figure 4.18  Locations on the forged steel crankshaft where the stress variation was traced over one complete cycle of the engine.

Figure 4.19  Locations on the cast iron crankshaft where the stress variation was traced over one complete cycle of the engine.
Figure 4.20 von Mises stress history (considering sign of principal stress) at different locations on the forged steel crankshaft at the engine speed of 2000 rpm.

Figure 4.21 von Mises stress history (considering sign of principal stress) at different locations on the cast iron crankshaft at the engine speed of 2000 rpm.
Figure 4.22 Comparison of maximum, minimum, mean, and range of stress at the engine speed of 2000 rpm at different locations on the forged steel crankshaft.

Figure 4.23 Comparison of maximum, minimum, mean, and range of stress at the engine speed of 2000 rpm at different locations on the cast iron crankshaft.
Figure 4.24  Variations of minimum stress, maximum stress, mean stress, and stress range at location 2 on the forged steel crankshaft as a function of engine speed.

Figure 4.25  Variations of minimum stress, maximum stress, mean stress, and stress range at location 2 on the cast iron crankshaft as a function of engine speed.
Figure 4.26  Effect of torsion on von Mises stress at location 2 at the engine speed of 2000 rpm for the forged steel crankshaft.

Figure 4.27  Effect of torsion on von Mises stress at location 2 at the engine speed of 2000 rpm for the cast iron crankshaft.
Figure 4.28  Effect of considering torsion in stresses at location 7 at different engine speeds for the forged steel crankshaft.

Figure 4.29  Effect of considering torsion in stresses at location 7 at different engine speeds for the cast iron crankshaft.
Figure 4.30  Locations on the crankshaft where strain gages were mounted.

Figure 4.31  Rainflow count of the von Mises stress with consideration of sign at location 2 at engine speed of 2000 rpm for the forged steel crankshaft.
Figure 4.32 Rainflow count of the von Mises stress with consideration of sign at location 2 at engine speed of 2000 rpm for the cast iron crankshaft.
5 Geometry, Material, and Cost Optimization

This chapter discusses the optimization options, their combination under a set of defined constraints and a comparison between the original forged steel crankshaft and the final optimized forged steel component. The main objective of this analysis was to optimize the weight and manufacturing cost of the forged steel crankshaft, which not only reduces the final production cost of the component, but also results in a lighter weight crankshaft which will increase the fuel efficiency of the engine.

Optimization carried out on this component is not the typical mathematical sense of optimization, because variables such as manufacturing and material parameters could not be organized in a mathematical function according to the set of constraints such that the maximum or minimum could be obtained. Instead, each optimization step was judged through the sense of engineering knowledge. In this case of optimization process, the final optimized geometry has definitely less weight than the original crankshaft but this does not mean that the weight could not be reduced further. In other words, this may not be the minimum possible weight under the set of constraints defined. As the main objective of this analysis, it was attempted to reduce the weight and final cost of the component.

The first step in the optimization process was weight reduction of the component considering dynamic loading, which means that the stress range under dynamic loading would not exceed the stress range magnitude in the original crankshaft. Possible weight
reduction options and their combinations were considered. Since the optimization and weight reduction was performed in an interactive manner and judged by manufacturing feasibility and cost, there is no guarantee that the obtained optimized crankshaft has the minimum weight possible. As a result, in this optimization problem, weight reduction by geometry optimization and other factors such as manufacturing process and material alternatives were considered separately.

The two main factors which were considered during optimization are stress range under dynamic load and bending stiffness. These factors were verified to be in the permissible limits. The optimization process was categorized in two stages; Stage I – No change in the engine block and the connecting rod, and Stage II – Some minor changes on the bearings and connecting rod. The optimization problem solving process is summarized in Figure 5.1, where it could be seen that the result from the first stage is input to the second stage. The general flow chart of the optimization process is shown in Figure 5.2. Objective function, design variables, and constraints are summarized in this figure and it is shown that the optimization process consisting of geometry, manufacturing process, and material optimization was performed simultaneously.

5.1 Component Specifications and Manufacturing Process

In order to carry out optimization process, it is necessary to have knowledge of the component dimensions, its service conditions, material of construction, manufacturing process, and other parameters that affect its cost. The service loading condition of this component was specified in Chapter 3. It was shown that the maximum
bending load occurs at the lowest operating engine speed. Therefore, this loading condition was considered as the primary loading condition for the optimization study.

The forged steel crankshaft material, AISI 1045, has the chemical composition shown in Table 5.1. As can be seen in this table, this high-strength low-alloy steel contains 0.45% carbon resulting in a yield strength of 625 MPa and fatigue strength of 359 MPa at $10^6$ cycles (Williams and Fatemi, 2007).

The main manufacturing process of the forged crankshaft is hot forging and machining and this is shown in a flowchart in Figure 5.3. Each step of this flowchart is described below, where the information about the forging and machining processes were obtained from the metal forming books and OEM websites.

1. The raw material samples of the AISI 1045 are inspected for chemical composition.
2. The material is shaped and cut to the rough dimensions of the crankshaft.
3. The shaped material is heated in the furnace to the temperature of $900^\circ C$ to $1100^\circ C$.
4. The forging process starts with the pre-forming dies, where the material is pressed between two forging dies to get a rough shape of the crankshaft.
5. The forging process continues with the forging of the pre-formed crankshaft to its first definite forged shape.
6. Trimming process cuts the flash which is produced and appears as flat unformed metal around the edge of the component.
7. The exact shape of the forged crankshaft is obtained in the coining process where the final blows of the hammer force the stock to completely fill every part of the finishing impression.
8. The final shaped forged crankshaft is now ready for the shot cleaning process. In this step the scales remained from forging process are removed.

9. The machining process starts with the facing and centering of the total length size. The facing process is a machining operation that is a form of turning in which the tool is fed at right angles to axis of workpiece rotation to produce flat surface. Centering refers to aligning the bearings of the component according to the final dimensions.

10. After alignment of all diameters the turning process will give a rough shape of cylinder to all cylindrical portions.

11. CAM turning is the process used to produce cylindrical components, typically on a lathe. A cylindrical piece of stock is rotated and a cutting tool is traversed along 2 axes of motion to produce precise diameters and depths. Turning can be either on the outside of the cylinder or on the inside (also known as boring) to produce tubular components to various geometries.

12. In the drilling operation, all inner diameters are drilled in the crankshaft geometry. The drilling mainly consists of oil holes.

13. Threads are cut on the inner surface of the bore at the back of the crankshaft and on the outer diameter of the front shaft.

14. Heat treatment is the next step to obtain the desired mechanical properties for the material.

15. Shot blasting consists of attacking the surface of a material with one of many types of shots. Normally this is done to remove scale from the surface.
16. Straightening process by application of external forces eliminates or reduces the curvatures, which can result by deformation during rolling, drawing, extrusion or due to non-uniform cooling. Thickness and geometry changes at different sections of a crankshaft cause non-uniform cooling during the forging process, which results in unwanted curvatures along the forged component.

17. After the straightening process the crankshaft is ready for grinding and being aligned to its final dimensions. Therefore, the grinding begins with the rough grinding of all diameters.

18. Since the crankshaft has eccentric cylinders the diameters have to be grinded using CAM.

19. The final grinding of diameters sets the cylinder diameters to their final acceptable tolerance. This is followed by grinding of other sections such as grooves using CAM.

20. The final step in grinding is face grinding, where the dimensions of the crankshaft will be finalized.

21. The last step in the machining process is balancing the crankshaft. In this process the crankshaft is mounted on two bearings in a device, and the dynamic balance of the component is checked. Mass and location of material removal is specified. Drilling holes in the counter weights will balance the crankshaft dynamically. The balance of the crankshaft is checked once more on the device.

22. Final washing of the component and preparation for final inspection.

23. The final inspection consists of checking diameter of cylinders, radius of crankshaft and distance of faces.
The finished forged crankshafts are sent for packing and dispatch.

As a main objective of the optimization study, the cost break down for each manufacturing step is very important. The influence of different parameters on the final cost of the component is another way for the cost break down. These parameters are shown in Table 5.2. Summarizing the manufacturing process to 12 steps as listed in Table 5.3 helps in cost estimation of the manufacturing process. In this table, each manufacturing step and its share in the total cost is shown. As can be seen for this table, about 50% of the final cost is machining process and 31% is material cost (Nallicheri et al., 1991).

5.2 Optimization Problem Definition

At the beginning of this chapter it was mentioned that the optimization study performed on this component was not the typical mathematical optimization process. There are different functions and limitations in the mathematical optimization, which are all defined as a set of variables. The main objective function is minimizing the weight, maximum stress at critical locations, and manufacturing cost. This function is subject to inequality constraints, equality constraints, and side constraints having the design variables as a tool for any optimization process. The bounded constraint in this component is maximum allowable stress of the material. The equality constraint is geometry limitations or fixed dimensions. And finally the design variables are upper and lower limit for size and geometry, material alternatives, and manufacturing processes.

Size optimization is an optimization approach, where the parameters do not change the overall shape of the component and only the size is modified. Geometrical
properties parameters such as thickness, diameter, and area are used as design variables in size optimization. There are other optimization methods such as Shape Optimization and Topological Optimization, which change the appearance of the geometrical domain.

The optimization approach in this study involved both size and shape optimizations. As discussed earlier, the optimization stages were considered not as a defined function of variables, but based on judgment using the results of the FEA and dynamic service load. The judgment was based on mass reduction, cost reduction, and improving fatigue performance using alternative materials and considering manufacturing aspects, as well as bending stiffness of the forged steel crankshaft. In Stage I, the design variables were thickness of the crank web, geometry of the cranks web, increasing inner hole diameters and their depth, and geometry changes on outer part of the crankpin bearing. Material alternatives and additional manufacturing operations such as fillet rolling were also investigated to reduce weight and cost as final steps of the first stage. Design variables for Stage II were considered to be changes in fillet radius, change in connecting rod bearing thickness, and bearing diameters. These geometry modifications are in addition to geometry modifications from Stage I.

The process of geometrical optimization in both stages was localized size and shape optimization with consideration of manufacturing limitations. Manufacturing process and material alternatives were other optimization variables that were considered for the optimization study. The algorithm of each optimization attempt is shown in Figure 5.2. Local geometrical optimization was performed in separate models, possible combinations of these optimizations were considered, and new models were created. Finite element analysis was performed on each model using the service load history
calculation from Chapter 3. Since this component is designed for many millions of cycles, the stresses generated from dynamic loading of the component are in the elastic region. Therefore, linear elastic analysis was considered to be sufficient for the optimization analysis. Manufacturing process and material alternatives were studied in a trial and error approach, after the geometrical optimization. In the manufacturing process optimization, the aim was to eliminate some steps to reduce the cost, while considering adding other steps in order to increase the performance of the crankshaft.

5.2.1 Objective Function

Objective function is defined as the parameters that are attempted to be optimized. In this study the weight, manufacturing cost and fatigue performance of the component were the main objectives. Optimization attempt was to reduce the weight and manufacturing cost, while improving the fatigue performance and maintaining the bending stiffness within permissible limits.

5.2.2 Constraints

Since the current crankshaft used in the engine has proper fatigue performance, optimization was carried out in such a way that the equivalent local stress amplitude at any location of the optimized model did not exceed the equivalent stress amplitude at the critical location of the original model. Considering two stages for the optimization approach, the following constraints were defined for each stage.

Stage 1 – Since the optimized crankshaft was expected to be interchangeable with the current one, the following dimensions were not changed:
• Outer diameters of different cylinders
• Crank radius
• Location of main bearings (distance between them)
• Geometry of main bearings
• Thickness and geometry of connecting rod bearing

**Stage II** – The first three constraints in Stage I remain the same, but the geometry of main bearings, and thickness and geometry of the connecting rod bearing could be modified.

It should be added that a main concern in crankshaft design is that the crankshaft has to be dynamically balanced, which could be achieved by maintaining the center of gravity on the center line and in the midway of both main bearings. This constraint has to always be fulfilled in the optimization process by changing the dimensions, and removing and/or adding material in the counter weights.

### 5.2.3 Design Variables

Parameters that could be changed during the optimization process are design variables. Considering the functions of the crankshaft and its constraints, the following design variables were considered in the optimization study:

**Stage I:**

• Thickness of crank web
• Geometry of crank web
• Increasing inner hole diameters and depths
• Geometry changes on the outer part of crankpin bearing
Stage II: Variables in stage I and

- Changes in fillet radii
- Change of connecting rod bearing length
- Change in the main bearing diameters

Manufacturing process and material alternatives are other design variables that were considered in this study. Since automotive crankshafts are mostly manufactured from microalloyed steels, this was considered as the alternative material. Microalloyed steels have the main advantage of eliminating the heat treatment step in the manufacturing process, which will reduce the cost of the final crankshaft. Other manufacturing aspects that are common in manufacturing of crankshafts including inducing compressive residual stress at the fillets were investigated to improve the fatigue performance of the component. This improvement would allow additional changes in the geometry in order to reduce the weight of the final optimized crankshaft.

5.3 Optimization Analysis Results and Discussion

In this section, details of each optimization stage is discussed, presenting the influence of each optimization potential on maximum stress range, bending stiffness, and weight difference with the original crankshaft.

5.3.1 Stage I – No Change in the Engine Block and the Connecting Rod Dimensions

In this stage of the optimization process, it is tried to reduce the weight and manufacturing cost without changing the geometry of the components that are connected
to the crankshaft. In addition, modification to manufacturing processes was considered to improve the fatigue performance and reduce the final cost of the component. Considering alternative material to improve the fatigue strength and reduce the cost was another concern of this stage. Substituting alternative materials allows manufacturing cost reduction by eliminating some manufacturing steps or modifying current production process.

5.3.1.1 Optimization Using Geometry Variables

Investigating the stress contour of the crankshaft FEA model during an engine cycle showed that some locations of the crankshaft experience lower stresses such as; counter weights and crank webs. The crankshaft has to be dynamically balanced and counter weights have a key role in balancing it. Therefore, although stresses applied to these sections are low, they can not be removed, but can only be changed according to other modifications made on the component. Since the whole engine rotating and moving mechanism, consisting of crankshaft, connecting rod, piston assembly, and flywheel, weight and inertia specifications have been considered in the design of the engine, mass reduction of one part could affect the response of the system to dynamic loads applied to the mechanism. Geometry optimization of the crankshaft results in weight reduction and therefore, reduction of the moment of inertia. In order to maintain the original dynamic behavior of the mechanism, the moment of inertia of the rotating components has to remain unchanged. Therefore, the flywheel moment of inertia has to be increased to cover the reduction made in the moment of inertia of the crankshaft, which results in weight increase of the flywheel. But since the flywheel radius is larger than the
crankshaft radius, weight increase of the flywheel is much less than weight reduction of the crankshaft.

Local shape optimization techniques were applied to different locations of the crankshaft to lower the weight of the crankshaft. After each optimization step the counter weights were balanced in order to have an accurate estimation of the weight reduction. Also, after each shape optimization iteration the optimized component was investigated for design feasibility and examined to find whether the design was the best feasible design possible. In a feasible design, the component stresses fall within the stress limits and bending stiffness does not change significantly, while its manufacturability is economical.

According to FE analysis, blue locations shown in Figure 5.4 have low stresses during service life and have the potential for material removal and weight reduction. Therefore, the following optimization options were considered. It should be added that the effect of mean stress in the results was negligible. Therefore, all stress concerns were about the stress range.

**Case 1: Increasing the depth of the drilled hole at the back of the crankshaft**

Since the dynamic balance is one of the main concerns in the optimization of this component as the first step it was tried to remove material symmetric to the central axis, which would not disturb the dynamic balance of the crankshaft. At the back of the crankshaft, as shown in Figure 5.5, there is a trenched hole. The depth of this bore does not affect the function of the crankshaft. Therefore, this hole could be drilled as far as possible in the geometry. Figure 5.6 shows the current depth and the increased depth of this bore at the back of the crankshaft. As could be seen in this figure, the oil hole
restricts further drilling. Since this hole is far from the boundary conditions and loadings on the crankshaft, stresses do not change at critical locations.

Figure 5.7 shows stress results for the critical location for different optimization cases, where value of one on the vertical axis of the fatigue stands for the original crankshaft (73 MPa for mean stress, 188 MPa for stress range, and 3.72 kg for weight). The slight change (1%) in stress range for Case 1 is a result of selection of the critical node, because each model is meshed separately and the node numbers and position slightly change. Therefore, each time a node is selected, a variation could be seen in the results.

Another concern of redesign and optimization process is bending stiffness. Bending stiffness of the crankshaft is important in the noise and performance of the component in the engine. In order to estimate the stiffness of the original crankshaft and compare it with other generated models, three locations with highest displacements were selected for stiffness investigation. These locations are shown in Figure 5.8. Chosen locations are located on the corners or exactly in the middle of a symmetric geometry. Therefore, there is a node located at these points, which always remains at the same location in each FE mesh. These nodes were used as selected nodes for checking the displacement. Figures 5.9 (a) and (b) show displacements in axial direction, \( U_3 \), and radial direction, \( \sqrt{U_1^2 + U_2^2} \), for the three locations on the crankshaft for different optimization potentials. As can be seen from this figure, the displacements do not change after the hole was drilled further at the back of the crankshaft. Considering this optimization process the weight of the crankshaft is reduces by 1%. 
Case 2: Increasing the hole diameter of the crankpin oil hole

Another optimization step which does not require any complicated changes in the geometry is increasing the hole diameter of the crankpin hole. Increasing the inner diameter of this hole will result in decreasing the moment inertia of the cross section. Therefore, in order to not increase the stress level at the fillet area, the fillet radius has to be increased. Increasing the fillet radius does not affect the connecting rod geometry since the current connecting rod has enough clearance. Applying these changes to the crankshaft causes the center of mass to move toward the counter weights. In order to balance the modified crankshaft, material has to be removed from the counter weight.

The optimized crankshaft in this step is shown in Figure 5.10. Considering this optimization option the stress range reduces by 5%, compared to the original model, as can be seen in Figure 5.7. The reason for this reduction is increasing the fillet area, which causes a lower stress concentration factor. As could be seen in Figure 5.9 (b) the displacement of three locations on the optimized crankshaft does not change more than 9% in radial direction. Displacement in U3 direction has slight changes of ±1% at locations A, B, and C. Weight reduction in this step is about 3%.

Case 3: Redesigning the geometry of the crankpin

Any weight reduction made on the crankpin geometry would result in material removal from the counter weights for dynamic balance of the crankshaft. Therefore, the next option considered for weight reduction was to redesign the crankpin geometry and remove material from this section. Material was cut from the outer geometry of the crankpin. Figure 5.11 shows the modified crankshaft after this geometry change. In this case, the stress range increased by 10%, which is shown in Figure 5.7. The increase of
stress was a result of higher stress concentration factor, which not only depends on the two diameters of the shafts and the fillet radius, but also depends on the thickness of the crank web, which was reduced in this case. Figure 5.12 shows a cross section view of the original and optimized crankshaft considering this case. It can be seen in this figure that the bending stiffness of the optimized cross section of the web has reduced as a result of material removal. Figures 5.9 (a) and (b) show the displacement of the critical locations on this optimization case. It can be seen that they increase no more than 10% in any direction. Weight reduction is 4% as a result of this optimization.

**Case 4: Rectangular material removal from the center of the crank web symmetric to the central axis**

Figure 5.13 shows the weight reduction option for this case. Material removal in the shape of rectangular was considered since the dynamic balance of the crankshaft would not be disturbed and further dynamic redesign of the counter weights were not necessary. The rectangle could be cut out from one side to the end of the other side, as far as the center of the rectangle remains centered to the central axis of the main bearings. The height of the rectangle is limited to the geometry of the crankpin bearing. It can be seen in Figure 5.13 that the upper side of the rectangle is restricted to the crankpin bearing. The depth of this rectangle was determined as it would not intersect with the oil hole drilled in the crank web. In addition, the moment of inertia of the cross section in the crank web should not be reduced significantly to avoid excess reduction of bending stiffness. This modification on the crank web did not increase the stress range and mean stress, but the stiffness of the crankshaft was changed. Displacement of different locations increase as a result of this optimization step, where location A in Figure 5.8 has
the highest displacement difference of 28% with the original crankshaft. Figures 5.9 (a) and (b) show the displacement of different locations in different directions. Though 28% may seem high, the main location and displacement direction which is important in the noise of the crankshaft is at location C and in the radial direction, which is about 8%. Weight reduction of about 3% is the result of this optimization step.

**Case 5: Semi-circle material removal from the center of the crank web symmetric to the central axis**

This step is a result of improvement of the previous step. Since the rectangular section depth is limited to the oil hole from the upper part, the middle section could be cut out more. Removing more material from the middle section of the crank web symmetric to the central axis resulted in cutting half circle from each part and reducing the weight more than 5%, in comparison with the original crankshaft. The optimized crankshaft from this optimization step is shown in Figure 5.14. Considering this optimization case the stresses slightly reduced with the applied dynamic load. This could be seen in Figure 5.7, where the mean stress and stress range are shown. As can be seen from Figures 5.9 (a) and (b), similar to the previous optimization case, the displacement of point C in radial direction is 8%.

**Case 6: Reducing the thickness of the web**

A high weight percentage of the crankshaft is in the crank web volume, therefore, reducing the weight of this section could result in a more efficient weight reduction of the component. Reducing the web thickness is another applied optimization case that was performed on the crankshaft web. Figure 5.15 shows this geometry change. As a result of this change in the crank web the center of gravity moves toward the crankpin bearing. In
order to dynamically balance the crankshaft, material has to be added to the counter weights. Adding material to the counter weights is possible by increasing their radius. This optimization option was restricted to the clearance between the piston and counter weights of the crankshaft. Since the current geometry is designed with specified clearance between counter weights and the piston, this optimization case could not be implemented without additional changes to the piston. In the next section it is shown that this case could be used in combination with other optimization cases. This optimization step did not change the stresses compared with the original crankshaft as could be seen in Figure 5.7. In addition, maximum displacement is about 16% at location A and in the U3 direction, which is in the same range of other optimization cases considered. Displacements at different locations in two directions are shown in Figures 5.9 (a) and (b). This optimization case results in the maximum weight reduction of 7%.

**Case 7: Modification of the crank web design**

Redesigning the crank web and removing material is the next optimization case that was applied to the crankshaft. Changing the design was with consideration of the manufacturing process. The final designed geometry should be feasible for the manufacturing process, which requires not having negative slopes. The crank web was modified such that no changes in the counter weights would be necessary. The result of this redesign is shown in Figure 5.16. Weight reduction is about 5% for this case and stresses remain the same as the original crankshaft. These are shown in Figure 5.7. The main advantage of this optimization process is that the displacements increase no more than 3% in comparison with the original crankshaft. This is a result of not changing the
crank web section, where bending moment is considered to be high. This could be seen in Figures 5.9 (a) and (b).

**Case 8: Eccentric crankpin hole**

This optimization step requires redesign of the outer geometry of the crankpin. By moving the centered oil hole toward the upper part of the crankshaft, where stresses are lower, the cross section of the hollow tube will have more area in the bottom, which is where higher stresses exist. With the increase of area in this section stresses would be lower, since the same bending moment is being applied to a wider area. Although stresses on the upper side will increase, the critical location still remains at the bottom part of the fillet. The following calculations show the effect of moving the oil hole on the nominal stress.

For the centered oil hole:

\[
I = \frac{\pi}{64}(D^4 - d^4) \quad (5.1)
\]

\[
\sigma_c = \frac{M \times D/2}{I} \quad (5.2)
\]

Considering the actual dimensions of the crankpin and oil hole of \(D = 33\) mm (1.3 in) and \(d = 16.9\) mm (0.665 in):

\[
\sigma_c = 3.04 \times 10^{-4} \times M \quad (5.3)
\]

where \(M\) is in N-mm and \(\sigma_c\) will be in MPa.

For the eccentric hole in the crankpin (based on nomenclature shown in Figure 5.17):

\[
l = \frac{D^3 / 2 - d^2(D / 2 + e)}{D^3 - d^2} \quad (5.4)
\]
\[ I = \left[ \frac{\pi D^4}{64} + \frac{\pi D^2}{4} (D/2 - l)^2 \right] - \left[ \frac{\pi d^4}{64} + \frac{\pi d^2}{4} (D/2 + e - l)^2 \right] \]  

(5.5)

\[ \sigma_e = \frac{M \times l}{I} \]  

(5.6)

where \( l \) is the distance to the centroid.

Substituting for values of \( D \) and \( d \), the final nominal stress at the bottom of the crankpin would be a function of \( e \). Figure 5.18 shows the variation of stress as a function of distance of center of the oil hole with the outer diameter, \( e \). The minimum stress occurs at \( e = 2.032 \) mm. Considering this figure and the calculations for the nominal stress, it can be calculated that the eccentricity reduces the stress for about 2.1\% at most. Since this percentage of stress reduction is small and this optimization step reduces the weight of the crankshaft by less than 1\%, this optimization step was not considered further.

**Combination options**

Since each optimization case was studied individually, further analysis is needed considering combination of these cases. Table 5.4 shows possible combination options, where “X” shows that the two cases in the row and the column could be used together on the crankshaft simultaneously and “-” indicates that the two cases can not be combined on the crankshaft geometry. Combination options have been considered as a redesigned crankshaft in which as many optimization cases as possible could be applied. For example, according to Table 5.4, Case 1 could be combined with any other optimization case, or Case 6 could be combined with Cases 1, 2, 3, and 7. These combinations were considered to develop the optimized crankshaft. Case 5 was not considered for combination with other cases because this case results in higher displacements and therefore, reduced stiffness for the crankshaft in comparison with other modifications to
the crank web. Cases 4, 6, and 7 have been developed for the crank web section, which are more efficient than Case 5 when combined together. In addition, forging the crank web geometry with semi-circular material removal could be difficult due to complexity of the geometry for the forging process. The optimized geometries resulting from each combination are shown in Figures 5.19 through 5.25.

FE models of possible combinations were created and FE analysis with dynamic load was considered for each combination. Results of stress range for these combination models are plotted in Figure 5.26 (a). It can be seen in this figure that the critical location (location 2) does not change as a result of geometry optimization. A comparison plot of stress ranges with the original stress ranges at the high stress locations of the current crankshaft are shown in Figure 5.26 (b), where the horizontal line at 1 stands for the original crankshaft. As could be seen in this figure, the first four combination options cause the stress range at the critically stressed location to increase. Referring to the previous analysis on separate optimization cases, it can be seen from Figure 5.7 that the only optimization option resulting in higher stress range than the original model is Case 3. Since the purpose of optimization is to at least maintain the original performance, this case was not considered as a local optimization option. Combinations of Cases 1, 2, and 6 and Cases 1, 2, and 7 are shown to have lower stress ranges than the original crankshaft. These combinations result in 12% and 9% weight reduction, as shown in Figure 5.27, respectively.

Combination of Cases 1, 2, and 6 has the same problem as considering Case 6 on its own. The counter weights radius is larger than the original crankshaft, which will cause interference between the counterweights and the piston. Combination of Cases 1, 2,
and 7 results in increased clearance between the counter weights and the piston, which means further weight reduction is possible. Replacing the material removal from the counter weights in combination 1, 2, and 7 will move the center of gravity toward the counter weights. Since reducing web thickness is not symmetric with respect to the central axis and results in more material removal from the side of the counter weights, the center of gravity will move toward the crankpin. At a certain web thickness reduction, the center of gravity will be located on the main axis, which means the crankshaft becomes dynamically balanced. The combination of Cases 1, 2, 6, and 7 results in 12% weight reduction in comparison with the original crankshaft, which is 3% more than combining Cases 1, 2, and 7. The reason for similar weight reduction between this combination and combination of Cases 1, 2, and 6 is that in Cases 1, 2, and 6 more material was removed from the crank web thickness and material was added to the counter weight to maintain the dynamic balance. But in Cases 1, 2, 6, and 7, it was not possible to reduce the crank web thickness further, since it was aimed not to add material and increase the counter weight radii due to the clearance between the piston and counter weights. As could be seen in Figure 5.27 the mean stress on the critical location of Cases 1, 2, 6, and 7 is the same as the original crankshaft and stress range is reduced slightly. Displacements of locations A, B, and C are not increased more than 15% in this combination of cases. Therefore, the stiffness of the optimized crankshaft is within the acceptable limit. Displacements at locations A, B, and C on different optimized crankshafts are shown in Figures 5.28 (a) and (b).
5.3.1.2 Modification to Manufacturing Process

As the next step for the optimization study it is tried to modify the production steps in order to reduce the cost or improve the performance of the current crankshaft. Further improvement of the performance could result in more geometry changes and weight reduction. The optimization in this section was investigated by considering adding compressive residual stress to the fillet area of the crankpin.

Due to lack of experimental information, the magnitude of the residual stress that could be induced in the studied crankshaft geometry is not identified. It was shown in the studies by Kamimura (1985), Park et al. (2001), and Chien et al. (2004), as discussed in the literature review Section 2.5, inducing compressive residual stress increases the fatigue strength of the crankshaft. Therefore, adding compressive residual stress on the fillet area of the current crankshaft increases its fatigue strength by 40% to 80% based on the material properties, crankshaft geometry, and applied rolling force.

Effect of nitriding as a surface hardening process was discussed in the literature review in Section 2.5. Since the nitriding process is time consuming in comparison with other heat treatment processes, it was not considered as a modification to manufacturing process to increase the performance of the crankshafts.

The effects of different surface hardening treatments such as quenching and tempering, ion-nitriding or fillet rolling on the fatigue properties were investigated in Pichard et al. (1993) research. Table 5.5 summarizes these effects. The fatigue bending moment for microalloyed 35MV7 steel without surface treatment was 1990 N.m. As could be seen in this table, the fatigue strength increases by 87% and 125% by fillet
rolling forces of 9 kN and 12 kN, respectively. For short nitriding treatments, the fatigue limit of microalloyed 35MV7 steel increased by about 135%.

5.3.1.3 Modification Using Alternatives Materials

One of the most common alternatives for the forged steel material is microalloyed steel. Pichard et al. (1993) performed a study on a microalloyed (MA) steel with titanium addition specially adapted for the production of forged crankshafts and which does not require any post-forging treatment. The use of MA steel enables elimination of any further heat treatment, resulting in shorter manufacturing process and consequently an increase in the forged crankshaft productivity. The metallurgical choice of this MA steel for crankshaft applications was based on the 35MV7 steel grade, with a typical composition of 0.35C, 1.8Mn, 0.25Si, 0.12V, and micro-addition of Ti. Based on the results of their research, 35MV7 control-cooled microalloyed steel shows similar tensile and rotating bending fatigue behavior as AISI 4142 quenched and tempered steel. In addition, the machinability of the microalloyed steel can be improved by about 40% in turning and about 160% in drilling (Pichard et al. 1993).

The effect of using different material with the same surface treatments are summarized in Table 5.5. As could be seen in this table, the quenched and tempered 1042 steel with short nitriding treatment has 56% higher fatigue strength than the quenched and tempered alloyed ductile iron with the same nitriding time. The quenched and tempered 32CDV13 steel with 7 hour nitriding time has the highest fatigue strength, which is about 49% higher than quenched and tempered 1042 steel with shorter nitriding time. Cost reduction of 13% is obtained for the final crankshaft by replacing the traditional AISI 4142 steel with 35MV7 control-cooled microalloyed steel. This includes
10% savings on the unfinished piece, 15% saving on mechanical operations and 15% saving on ion nitriding treatment (Pichard et al. 1993).

A comparison between the material properties used in the current crankshaft, AISI 1045 steel, and microalloyed steel 35MV7 indicates similar yield strengths, 12% higher tensile strength, and higher fatigue strength (by 21%) at $10^6$ cycles for the microalloyed steel, as summarized in Table 5.6. Further study on the cost of final product is discussed in the cost analysis section.

5.3.1.4 Final Optimized Crankshaft from Stage I

Considering the manufacturing processes, the geometry of the crankshaft could be modified even more to take advantage of the results of improved fatigue strength due to fillet rolling and/or use of microalloyed steel. Further modification to the crankpin geometry is possible. Increasing the crankpin hole is an option which does not influence the manufacturing process and is not an expensive process. Increasing the hole diameter from 18.3 mm (0.72 inch) to 25.4 mm (1 inch) and reducing the crank web thickness in order to maintain dynamic balance of the crankshaft, will cause the stress range at the critical location to increase by 7%, which could be covered by the beneficial effect of the compressive residual stress from fillet rolling. This modification is shown as Case 1, 2*, 6*, and 7 in Figures 5.26 through 5.28. Since the wall thickness in the crankpin area is limited, further increasing the hole diameter to larger than 25.4 mm was not possible, because sufficient material is needed to restrict plastic deformation in the rolling process to result in residual stress. This modification reduces the weight of the original crankshaft by 18%. As discussed in Section 5.3.1.1, weight reduction of the crankshaft results in reduction of moment of inertia of the component, which is about 21% for the final
optimized geometry. Therefore, in order to maintain the total moment of inertia of the rotating parts mass is added to the flywheel to increase its moment of inertia. Weight increase of the flywheel is less than weight reduction of the crankshaft, because the flywheel radius is larger than the crankshaft radius. The use of microalloyed steel will reduce the final cost by eliminating the heat treatment process and increasing the machinability of the crankshaft. In addition, the dynamic balance is maintained during the optimization process and the bending stiffness is verified to be in the permissible limits.

The modified dimensions of the original forged steel crankshaft and the final optimized geometry are shown in Figure 5.29. The geometry optimization process is summarized in a flowchart shown in Figure 5.30.

5.3.2 Stage II - Limited Changes to Connecting Rod and Bearings/Engine Block

Stage II of optimization includes geometry optimization with limited changes in the diameter of the main bearing, crankpin bearing length, and fillet radius of crankpin and main bearings. It is aimed in this stage to obtain a lighter crankshaft that further reduces the material and manufacturing costs.

For this optimization stage, the optimized crankshaft from Stage I was redesigned to obtain higher fatigue performance and reduced weight. Reducing the main bearing diameters was the first step of geometry optimization in this stage. Since the critical location is in the fillet area of the crankpin, reduction in the main bearing diameters does not affect the stresses in the critical location, but increases the stresses in the fillet area between the main bearings and the crank web attachment. Increased stress ranges in this
section were admissible, because the stress ranges were lower than the operating critical stress range in the existing crankshaft, defined as and equal to 210 MPa at location 2. Figure 5.31 shows the redesigned model with modification on the main bearings, and Figure 5.32 shows dimensions on the original crankshaft and optimized crankshafts from Stages I and II. This modification will result in 10% weight reduction in comparison with the optimized crankshaft from the previous step, and 26% reduction in comparison with the original component. Stress range results at different locations of this optimized crankshaft are shown in Figure 5.33. Stress ranges at locations 1 and 3 increase by 85% and 43%, respectively, but the critical location remains location number 2. The increase of 10% in stress range at location 2 is easily compensated for by residual stress from fillet rolling. This optimization will reduce the stiffness of the component significantly. Displacement in the axial direction, U3, is about 47% for location B in Figure 5.8, and radial displacement of location C is 46%. These are shown in Figure 5.34. In order to implement this optimization, thrust bearings have to be used in the engine to support both sides of the crankshaft. Thrust bearings will limit the axial displacement of the crankshaft resulting in increase of stiffness and reduction of the longitudinal displacement, which may allow additional changes and further weight reduction of the crankshaft, connecting rod, and piston assembly. This requires more analysis and modeling. In addition, the cost of angular contact ball bearing which can carry both longitudinal and radial load is higher than the cost of single groove ball bearing, currently used in the engine.

Changing the crankpin fillet radius was not considered as a part of this optimization stage since the radius has been increased in Stage I without requiring any changes to the connecting rod geometry. In addition, increasing the fillet radius in Stage I
caused the stresses to decrease, in comparison with the original model. Therefore, no more change was required in this regard, to increase the fatigue performance of the crankshaft at this location.

Decrease in the crankpin bearing length was considered and the result was increase in the weight of the component. Reducing the length of the crankpin bearing required machining less material from the crankpin of the as-forged crankshaft. Also, to balance the crankshaft, material had to be added to the counter weights. Therefore, the final optimized crankshaft weighed heavier than the modified crankshaft from Stage I. If reducing crankpin bearing length is not performed by machining less material from the crankpin, the distance between crank webs has to be reduced. This will cause the crankshaft to shorten in length. This means that the final optimized crankshaft in this case will be shorter and smaller than the original one. The objective of the optimization on this component was that the final optimized crankshaft could be used in the current engine, with limited modifications to the bearings and/or connecting rod, but not changing the size of the entire engine block. Based on these considerations, the final optimized geometry from this stage resulted in reduced main bearing diameters of the crankshaft. The optimized crankshaft from this stage requires changing the main bearings to thrust bearings with proper size according to optimized diameters. As discussed earlier, this requires further analysis which could result in additional changes to the crankshaft, connecting rod, and piston assembly.
5.4 Cost Analysis

Cost analysis is based on geometry changes and weight, modification in manufacturing process and the use of alternative material. The optimized geometry requires redesign and remanufacturing the forging dies used. The geometry parameters that influence machining and the final cost of the component include the increase of drilling process, because the drilled holes at the back of the crankshaft and the crankpin are redesigned to have larger diameters, and the bore at the back is modified to have more depth than the original bore.

Adding residual stress by fillet rolling process is a parameter in the manufacturing process that will add to the cost of the finished component.

Nallicheri et al. (1991) studied material alternatives for the automotive crankshaft based on manufacturing economics. The approach they used in technical cost modeling is to separate the different cost elements and estimate each one separately. As defined in their study, the costs are separated in two elements of variable and fixed costs. Variable cost elements are the contribution to piece cost whose values are independent of the number of elements produced. And in contrast with the variable costs, the fixed costs are those elements of piece cost which are a function of the annual production volume. Assumptions made for the cost analysis modeling are tabulated in Tables 5.7 and 5.8. These tables show manufacturing assumptions used as the basis of cost estimation for a 32.6 lb crankshaft using hot rolled steel bar and microalloyed steel. The major difference between the assumptions in the case of hot rolled steel forging and microalloy forging were those of the die life and material cost. A minor improvement in die life was assumed owing to the enhanced formability of microalloyed steel. In case of microalloy
forging, the quench and temper step was eliminated, resulting in cost savings. The fabrication costs estimated by different models for the two cases are presented in Table 5.9. The use of microalloy forging eliminates some heat treating costs and yields savings in the as-forged state over conventional steel forgings. Figure 5.35 shows the finished piece costs for alternative manufacturing processes as a function of annual production volume.

Although microalloy grade steel, as of March 2007, is $0.028/lb more expensive than hot-rolled steel bar, the heat treatment cost savings, which are at least $0.15/lb, are large enough to offset this difference (Wicklund, 2007). Apart from the heat treatment costs, the use of microalloyed steel also results in savings in machining costs stemming from enhanced production rates and longer tool life (Nallicheri et al. 1991). In addition, microalloyed steel has 5% to 10% better machinability than quenched and tempered steel (Wicklund, 2007). Considering these factors, along with the reduced material cost due to the 18% weight reduction for Stage I and 26% for Stage II, indicates significant reduction in the total cost of the forged steel crankshaft. It should, however be mentioned that Stage II requires higher cost angular contact ball bearing and additional analysis is needed for this stage, which may result in additional changes in the crankshaft, connecting rod, and piston assembly. Figure 5.36 shows the modified manufacturing process for the optimized crankshaft considering aforementioned changes.
Table 5.1  Chemical composition of AISI 1045 by percent weight (Williams and Fatemi, 2006).

<table>
<thead>
<tr>
<th>Element</th>
<th>Percent by weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>0.45</td>
</tr>
<tr>
<td>Mn</td>
<td>0.81</td>
</tr>
<tr>
<td>P</td>
<td>0.016</td>
</tr>
<tr>
<td>S</td>
<td>0.024</td>
</tr>
<tr>
<td>Si</td>
<td>0.27</td>
</tr>
<tr>
<td>Al</td>
<td>0.033</td>
</tr>
<tr>
<td>Cr</td>
<td>0.1</td>
</tr>
<tr>
<td>Ni</td>
<td>0.05</td>
</tr>
<tr>
<td>Cu</td>
<td>0.13</td>
</tr>
<tr>
<td>N</td>
<td>0.008</td>
</tr>
<tr>
<td>O</td>
<td>13 ppm</td>
</tr>
<tr>
<td>Fe</td>
<td>balance</td>
</tr>
</tbody>
</table>

Table 5.2  Cost breakdown for a forged steel crankshaft weighing 32.6 lbs by factor in the study by (Nallicheri, 1991).

<table>
<thead>
<tr>
<th>By Factor</th>
<th>$/part</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Materials</td>
<td>$14.11</td>
<td>31.00%</td>
</tr>
<tr>
<td>Tool</td>
<td>$1.37</td>
<td>3.00%</td>
</tr>
<tr>
<td>Energy</td>
<td>$1.34</td>
<td>2.94%</td>
</tr>
<tr>
<td>Labor</td>
<td>$13.75</td>
<td>30.21%</td>
</tr>
<tr>
<td>Capital</td>
<td>$10.97</td>
<td>24.10%</td>
</tr>
<tr>
<td>Maintenance</td>
<td>$2.48</td>
<td>5.45%</td>
</tr>
<tr>
<td>Tax</td>
<td>$0.74</td>
<td>1.63%</td>
</tr>
<tr>
<td>Insurance</td>
<td>$0.62</td>
<td>1.36%</td>
</tr>
<tr>
<td>Building</td>
<td>$0.29</td>
<td>0.65%</td>
</tr>
<tr>
<td>Credit</td>
<td>($0.16)</td>
<td>-0.35%</td>
</tr>
<tr>
<td>TOTAL</td>
<td>$45.51</td>
<td>100.00%</td>
</tr>
</tbody>
</table>
Table 5.3  Cost breakdown for forged steel crankshaft weighing 32.6 lbs by process in the study by (Nallicheri, 1991).

<table>
<thead>
<tr>
<th>By Process</th>
<th>$/part</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Building</td>
<td>$0.29</td>
<td>0.6%</td>
</tr>
<tr>
<td>Raw Material</td>
<td>$13.49</td>
<td>29.6%</td>
</tr>
<tr>
<td>Heating</td>
<td>$3.36</td>
<td>7.4%</td>
</tr>
<tr>
<td>Sizing</td>
<td>$0.62</td>
<td>1.4%</td>
</tr>
<tr>
<td>Pre-forming</td>
<td>$1.20</td>
<td>2.6%</td>
</tr>
<tr>
<td>Forging</td>
<td>$1.89</td>
<td>4.2%</td>
</tr>
<tr>
<td>Trimming</td>
<td>$0.48</td>
<td>1.0%</td>
</tr>
<tr>
<td>Coining</td>
<td>$0.16</td>
<td>0.4%</td>
</tr>
<tr>
<td>Heat Treating</td>
<td>$1.58</td>
<td>3.5%</td>
</tr>
<tr>
<td>Shot Blast</td>
<td>$0.13</td>
<td>0.3%</td>
</tr>
<tr>
<td>Inspection</td>
<td>$0.87</td>
<td>1.9%</td>
</tr>
<tr>
<td>Machining</td>
<td>$21.43</td>
<td>47.1%</td>
</tr>
<tr>
<td>TOTAL</td>
<td>$45.51</td>
<td>100.0%</td>
</tr>
</tbody>
</table>

Table 5.4  Possible combination of optimization cases*.

<table>
<thead>
<tr>
<th></th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
<th>Case 4</th>
<th>Case 5</th>
<th>Case 6</th>
<th>Case 7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>-</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Case 2</td>
<td>X</td>
<td>-</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Case 3</td>
<td>X</td>
<td>X</td>
<td>-</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Case 4</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Case 5</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Case 6</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>X</td>
</tr>
<tr>
<td>Case 7</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>-</td>
<td>-</td>
<td>X</td>
<td>-</td>
</tr>
</tbody>
</table>

* For example Case 1 can be combined with all other cases and Case 6 can be combined with Cases 1, 2, 3, and 7.
Table 5.5  Fatigue experiment results on specimens from competitor crankshaft materials (Pichard et al., 1993).

<table>
<thead>
<tr>
<th>MATERIAL</th>
<th>STATE</th>
<th>SURFACE HARDENING TREATMENT</th>
<th>BENDING MOMENT (N.m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ductile iron</td>
<td>As cast</td>
<td>Without</td>
<td>± 1500</td>
</tr>
<tr>
<td>Alloyed ductile iron</td>
<td>Q. + Temp.</td>
<td>Ion nitriding : 4 h</td>
<td>± 2230</td>
</tr>
<tr>
<td>Ductile iron</td>
<td>As cast</td>
<td>Fillet rolling. P (↑: 8000 N)</td>
<td>± 2935</td>
</tr>
<tr>
<td>1042 steel</td>
<td>Q. + Temp.</td>
<td>Ion nitriding : 4 h</td>
<td>± 3480</td>
</tr>
<tr>
<td>35 MV7 steel</td>
<td>Cont. Cooled</td>
<td>Ion nitriding : 4 h</td>
<td>± 4660</td>
</tr>
<tr>
<td>32 CDV13 steel</td>
<td>Q. + Temp.</td>
<td>Ion nitriding : 7 h</td>
<td>± 5170</td>
</tr>
</tbody>
</table>

(*) P = Pressure

Table 5.6  Typical mechanical and fatigue properties of Ti-added controlled-cooled 35MV7 steel (Pichard et al., 1993) and AISI 1045 steel (Williams and Fatemi, 2007).

<table>
<thead>
<tr>
<th>Steel</th>
<th>Heat Treatment</th>
<th>Ultimate Strength (MPa)</th>
<th>Yield Strength (MPa)</th>
<th>Fatigue Strength (MPa)</th>
<th>Percent Elongation (%)</th>
<th>Percent Reduction in Area (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AISI 1045</td>
<td>Q + T</td>
<td>827</td>
<td>625</td>
<td>395</td>
<td>54</td>
<td>58</td>
</tr>
<tr>
<td>35MV7</td>
<td>Cont. Cooled</td>
<td>925</td>
<td>630</td>
<td>478</td>
<td>15</td>
<td>50</td>
</tr>
</tbody>
</table>

Table 5.7  Manufacturing assumptions for a forged steel crankshaft weighing 32.6 lbs (Nallicheri, 1991).

<table>
<thead>
<tr>
<th>Part Weight</th>
<th>32.6 lbs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Material</td>
<td>Hot Rolled Steel Bar (4130)</td>
</tr>
<tr>
<td>Die Life (parts)</td>
<td>15,000</td>
</tr>
<tr>
<td>Material Cost</td>
<td>$0.22/lb</td>
</tr>
<tr>
<td>Forging Impressions</td>
<td>2</td>
</tr>
<tr>
<td>Cavities/Impression</td>
<td>1</td>
</tr>
<tr>
<td>Maintenance</td>
<td>4%</td>
</tr>
<tr>
<td>Building Area</td>
<td>25,000 sq. ft.</td>
</tr>
</tbody>
</table>
Table 5.8 Manufacturing assumptions for a microalloyed steel forging crankshaft weighing 32.6 lbs (Nallicheri, 1991).

<table>
<thead>
<tr>
<th>Part Weight</th>
<th>32.6 lbs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Material</td>
<td>Microalloy Steel</td>
</tr>
<tr>
<td>Die Life (parts)</td>
<td>20,000</td>
</tr>
<tr>
<td>Material Cost</td>
<td>$0.25/lb</td>
</tr>
<tr>
<td>Forging Impressions</td>
<td>2</td>
</tr>
<tr>
<td>Cavities/Impression</td>
<td>1</td>
</tr>
<tr>
<td>Maintenance</td>
<td>4%</td>
</tr>
<tr>
<td>Building Area</td>
<td>25,000 sq. ft.</td>
</tr>
</tbody>
</table>

Table 5.9 Break of cost by factor for a crankshaft weighing 32.6 lbs, fully machined part cost (Nallicheri, 1991).

<table>
<thead>
<tr>
<th>Cast</th>
<th>Forged</th>
<th>µalloy</th>
<th>ADI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variable Costs</td>
<td>$26.79</td>
<td>$28.08</td>
<td>$26.56</td>
</tr>
<tr>
<td>Fixed Costs</td>
<td>$12.04</td>
<td>$17.40</td>
<td>$11.60</td>
</tr>
<tr>
<td>Total Costs</td>
<td>$38.83</td>
<td>$45.48</td>
<td>$41.15</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Cast</th>
<th>Forged</th>
<th>µalloy</th>
<th>ADI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Forming Cost</td>
<td>$23.26</td>
<td>$23.74</td>
<td>$22.83</td>
</tr>
<tr>
<td>Machining Cost</td>
<td>$15.56</td>
<td>$21.74</td>
<td>$18.32</td>
</tr>
<tr>
<td>Total Costs</td>
<td>$38.83</td>
<td>$45.48</td>
<td>$41.15</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Cast</th>
<th>Forged</th>
<th>µalloy</th>
<th>ADI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Materials</td>
<td>$17.44</td>
<td>$13.99</td>
<td>$15.06</td>
</tr>
<tr>
<td>Tooling</td>
<td>$2.76</td>
<td>$3.54</td>
<td>$2.90</td>
</tr>
<tr>
<td>Labor</td>
<td>$6.69</td>
<td>$12.54</td>
<td>$10.33</td>
</tr>
<tr>
<td>Capital</td>
<td>$7.33</td>
<td>$10.34</td>
<td>$8.73</td>
</tr>
<tr>
<td>Maintenance</td>
<td>$1.72</td>
<td>$2.27</td>
<td>$1.91</td>
</tr>
<tr>
<td>Other</td>
<td>$2.88</td>
<td>$2.89</td>
<td>$2.22</td>
</tr>
<tr>
<td>Total</td>
<td>$38.83</td>
<td>$45.48</td>
<td>$41.15</td>
</tr>
</tbody>
</table>
Forged Steel Crankshaft Optimization Alternatives

Focus:
- Minimize mass while maintaining the overall shape and size of crankshaft, connecting rod and engine block
- Modify manufacturing process
- Considering microalloyed steel as material alternative

No Change in Connecting Rod and Engine Block

STAGE I

Limited Change in Connecting Rod and Engine Block

STAGE II

Focus:
- Minimize mass and increase performance with limited changes in engine block and/or connecting rod geometry

Figure 5.1 Crankshaft optimization stages followed in this study
Figure 5.2  General flow chart of forged steel crankshaft optimization procedure.
Figure 5.3 Manufacturing process flowchart for the forged steel crankshaft.
Figure 5.4  Stress distribution under critical loading condition at the crank angle of 355 degrees.

Figure 5.5  Location of the threaded hole at the back of the crankshaft.
Figure 5.6 Case 1: Original hole depth and increased hole depth. Dimensions are in mm.

Figure 5.7 Stress results for the critical location. The horizontal line at 1 stands for the original crankshaft with 73 MPa for mean stress, 188 MPa for stress range, and 3.72 kg for weigh. Case 1: Increasing the depth of the drilled hole at the back of the crankshaft, Case 2: Increasing the hole diameter of the crankpin oil hole and increasing the fillet radius, Case 3: Redesigning the geometry of the crankpin, Case 4: Rectangular material removal from the center of the crank web symmetric to the central axis, Case 5: Half circle material removal from the center of the crank web symmetric to the central axis, Case 6: Reducing the thickness of the web, and Case 7: Modification to the crank web design.
Figure 5.8  Locations on the forged crankshaft where displacements were traced over one complete engine cycle.
Figure 5.9 Displacements in (a) Longitudinal direction, U3, (b) Radial direction, resultant of U1 and U2 ($\sqrt{U1^2 + U2^2}$), at different locations on the crankshaft shown in Figure 5.8. Case 1: Increasing the depth of the drilled hole at the back of the crankshaft, Case 2: Increasing the hole diameter of the crankpin oil hole and increasing the fillet radius, Case 3: Redesigning the geometry of the crankpin, Case 4: Rectangular material removal from the center of the crank web symmetric to the central axis, Case 5: Half circle material removal from the center of the crank web symmetric to the central axis, Case 6: Reducing the thickness of the web, and Case 7: Modification to the crank web design.
Figure 5.10  Case 2: Original crankpin hole diameter and increased diameter with modification to the counter weights for dynamic balance. Including increased fillet radii. All dimensions are in mm.

Figure 5.11  Case 3: Redesigned crankpin geometry and modification to counter weights for dynamic balance.
Figure 5.12 Cross section view of the original and optimized crankshaft in Case 3.

Figure 5.13 Case 4: Rectangular material removal from the crank web symmetric to the central axis of the crankshaft.
Figure 5.14 Case 5: Semi-circular material removal from the crank web symmetric to the central axis of the crankshaft.

Figure 5.15 Case 6: Web thickness reduction and material added to the counter weights for dynamic balance of the crankshaft.
Figure 5.16  Case 7: Modification to crank web geometry.

Figure 5.17  Dimensions of eccentricity of the crankpin hole.
Figure 5.18  Effect of eccentricity on bending moment (M) coefficient in stress
\[ \sigma_e = \frac{M \times l}{I} . \]

Figure 5.19  Optimized crankshaft with combination of Cases 1, 2, 3, and 4.
Figure 5.20  Optimized crankshaft with combination of Cases 1, 2, 3, and 6.

Figure 5.21  Optimized crankshaft with combination of Cases 1, 2, 3, and 7
Figure 5.22  Optimized crankshaft with combination of Cases 1, 2, 3, 4, and 6.

Figure 5.23  Optimized crankshaft with combination of Cases 1, 2, and 6.
Figure 5.24 Optimized crankshaft with combination of Cases 1, 2, and 7.

Figure 5.25 Optimized crankshaft with combination of Cases 1, 2, 6, and 7.
Figure 5.26 (a) Stress range results for different combination options at different locations on the crankshaft. (b) Comparison of stress range results with the original crankshaft at different locations. The horizontal line at 1 stands for the original crankshaft. Case 1: Increasing the depth of the drilled hole at the back of the crankshaft, Case 2: Increasing the hole diameter of the crankpin oil hole and increasing the fillet radius, Case 3: Redesigning the geometry of the crankpin, Case 4: Rectangular material removal from the center of the crank web symmetric to the central axis, Case 6: Reducing the thickness of the web, Case 7: Modification to the crank web design, Case 2*: Increasing the hole diameter of the crankpin oil hole further than Case 2, and Case 6*: Reducing the thickness of the web further than Case 6.
Figure 5.27 Stress results for the critical location and weights. The horizontal line at 1 stands for the original crankshaft with 73 MPa for mean stress, 188 MPa for stress range, and 3.72 kg for weigh. Case 1: Increasing the depth of the drilled hole at the back of the crankshaft, Case 2: Increasing the hole diameter of the crankpin oil hole and increasing the fillet radius, Case 3: Redesigning the geometry of the crankpin, Case 4: Rectangular material removal from the center of the crank web symmetric to the central axis, Case 6: Reducing the thickness of the web, Case 7: Modification to the crank web design, Case 2*: Increasing the hole diameter of the crankpin oil hole further than Case 2, and Case 6*: Reducing the thickness of the web further than Case 6.
Figure 5.28  Displacements in (a) Longitudinal direction, U3, (b) Radial direction, resultant of U1 and U2 ($\sqrt{U1^2 + U2^2}$), at different locations on the optimized crankshaft in directions shown in Figure 5.8. Case 1: Increasing the depth of the drilled hole at the back of the crankshaft, Case 2: Increasing the hole diameter of the crankpin oil hole and increasing the fillet radius, Case 3: Redesigning the geometry of the crankpin, Case 4: Rectangular material removal from the center of the crank web symmetric to the central axis, Case 6: Reducing the thickness of the web, Case 7: Modification to the crank web design, Case 2*: Increasing the hole diameter of the crankpin oil hole further than Case 2, and Case 6*: Reducing the thickness of the web further than Case 6.
Figure 5.29  Dimensions of the final optimized geometry, which is a combination of Cases 1, 2, 6, and 7. Case 1: Increasing the depth of the drilled hole at the back of the crankshaft, Case 2: Increasing the hole diameter of the crankpin oil hole and increasing the fillet radius, Case 6: Reducing the thickness of the web, and Case 7: Modification to the crank web design. All dimensions shown are in mm.
Figure 5.30  Geometry optimization flowchart.
Figure 5.31   Optimized crankshaft from Stage II of the optimization process.
Figure 5.32 Dimensions of the optimized geometry from Stage II, which is a combination of Cases 1, 2, 6, and 7 including changes from Stage II as compared with Stage I and original crankshaft. Case 1: Increasing the depth of the drilled hole at the back of the crankshaft, Case 2: Increasing the hole diameter of the crankpin oil hole and increasing the fillet radius, Case 6: Reducing the thickness of the web, and Case 7: Modification to the crank web design. All dimensions shown are in mm.
Figure 5.33  Stress range at different locations in the original and optimized crankshaft from Stage II.
Figure 5.34  Displacements in (a) Longitudinal direction, $U_3$, and (b) Radial direction, resultant of $U_1$ and $U_2$ ($\sqrt{U_1^2 + U_2^2}$), at different locations on the optimized crankshaft from Stage II in directions shown in Figure 5.8.
Figure 5.35  Finished piece costs – alternative processes as a function of annual production volume for 32.6 lbs automotive crankshafts (Nallicheri et al., 1991).
Figure 5.36  Modified manufacturing process for the forged steel crankshaft.
6 Summary and Conclusions

A forged steel and a ductile cast iron crankshaft were chosen for this study, both of which belong to similar single cylinder four stroke air cooled gasoline engines. First, both crankshafts were digitized using a CMM machine. Load analysis was performed based on dynamic analysis of the slider crank mechanism consisting of the crankshaft, connecting rod, and piston assembly, using analytical approach and verification of results by ADAMS modeling of the engine. FEA model of each crankshaft was created and superposition of stresses from unit load analysis in the FEA, according to dynamic loading, resulted in stress history at different locations on the crankshaft geometry during an entire engine cycle. As the next step of this study, geometry and manufacturing cost optimization was performed on the forged steel crankshaft. In the first stage of geometry optimization local geometry changes at different locations on the crankshaft were considered. Final optimized geometry from the first stage, which is replaceable in the engine without any change to the engine block and the connecting rod, is a result of combining local geometry optimization potentials considering manufacturing feasibility and cost. In the next stage of optimization, minor changes to the engine block and/or connecting rod geometry was considered.

The following conclusions can be drawn from the analysis conducted in this study:
1. Dynamic loading analysis of the crankshaft results in more realistic stresses whereas static analysis provides overestimated results. Accurate stresses are critical input to fatigue analysis and optimization of the crankshaft.

2. There are two different load sources in an engine; inertia and combustion. These two load sources cause both bending and torsional load on the crankshaft. The maximum load occurs at the crank angle of 355 degrees for this specific engine. At this angle only bending load is applied to the crankshaft.

3. Considering torsional load in the overall dynamic loading conditions has no effect on von Mises stress at the critically stressed location. The effect of torsion on the stress range is also relatively small at other locations undergoing torsional load. Therefore, the crankshaft analysis could be simplified to applying only bending load.

4. Superposition of FEM analysis results from two perpendicular loads is an efficient and simple method of achieving stresses for different loading conditions according to forces applied to the crankshaft from the dynamic analysis.

5. Experimental stress and FEA results showed close agreement, within 7% difference. These results indicate non-symmetric bending stresses on the crankpin bearing, whereas using analytical method predicts bending stresses to be symmetric at this location. The lack of symmetry is a geometry deformation effect, indicating the need for FEA modeling due to the relatively complex geometry of the crankshaft.

6. Critical (i.e. failure) locations on the crankshaft geometry are all located on the fillet areas because of high stress gradients in these locations, which result in high stress concentration factors.
7. Using the rainflow cycle counting method on the critical stress history plot shows that in an entire cycle only one peak is important and can cause fatigue damage in the component.

8. Geometry optimization resulted in 18% weight reduction of the forged steel crankshaft, which was achieved by changing the dimensions and geometry of the crank webs while maintaining dynamic balance of the crankshaft. This stage of optimization did not require any changes in the engine block or connecting rod.

9. As a result of geometry optimization from Stage II, the weight of the crankshaft was reduced by 26%. Crankshaft geometry changes in this optimization stage required changing the main bearings in the engine according to the optimized diameters and using thrust bearings to reduce the increase of axial displacement of the crankshaft.

10. Adding fillet rolling was considered in the manufacturing process. Fillet rolling induces compressive residual stress in the fillet areas, which results in 165% increase in fatigue strength of the crankshaft and increases the life of the component significantly.

11. Using microalloyed steel as an alternative material to the current forged steel results in the elimination of the heat treatment process. In addition, considering better machinability of the microalloyed steel along with the reduced material cost due to the 18% weight reduction in Stage I or the 26% weight reduction in Stage II result in significant reduction in overall cost of the forged steel crankshaft. But further analysis and modeling is needed for the 26% weight reduction in Stage II, due to requiring thrust bearing after optimization, which may result in changes to other components.


http://www.makinamuhendisi.com/, (accessed April, 2007)


Appendix A

Analytical Approach to Dynamic Analysis of the Crankshaft, Connecting Rod, and Piston Assembly Mechanism (Slider Crank Four Bar Linkage).

The following analysis is required in order to obtain the load history applied to crankshaft bearings. These calculations are based on the single degree of freedom mechanism shown in Figure A.1.

The angular velocity and acceleration of the crankshaft are given by:

$$\omega_1 = \frac{d\theta}{dt}$$

(1)
\[ \alpha = \frac{d\omega_1}{dt} \]  

(2)

The angle of the connecting rod, \( \beta \), is related to \( \theta \) by:

\[ \sin(\beta) = \frac{L_1 \sin(\theta)}{L_2} \]  

(3)

Angular velocity and acceleration of the connecting rod are obtained by differentiating the angle \( \beta \) and \( \omega_2 \) with respect to time, respectively:

\[ \omega_2 = \frac{d\beta}{dt} \]  

(4)

\[ \alpha_2 = \frac{d\omega_2}{dt} \]  

(5)

Differentiating Equation (3) with respect to time will result in:

\[ \omega_2 \cos(\beta) = \frac{\omega_1 L_1 \cos(\theta)}{L_2} \]  

(6)

Calculating \( \cos(\beta) \) from Equation (3):

\[ \cos(\beta) = \sqrt{1 - \frac{L_1^2 \sin^2(\theta)}{L_2^2}} \]  

(7)

Replacing Equation (7) into Equation (6) gives the angular velocity of the connecting rod, \( \omega_2 \), as a function of \( \theta \):

\[ \omega_2 = \frac{\omega_1 L_1 \cos(\theta)}{L_2 \sqrt{1 - \frac{L_1^2 \sin^2(\theta)}{L_2^2}}} \]  

(8)

The angular acceleration of the connecting rod, \( \alpha_2 \), is calculated by differentiating Equation (8) with respect to time and replacing for \( \frac{d\omega_1}{dt} \) and \( \frac{d\theta}{dt} \) from Equation (2) and Equation (1), respectively:
Location of center of mass of the connecting rod, \( r_g \), from the origin, A, is given in X and Y directions by the following equations:

\[
\begin{align*}
    r_{gx} &= L_1 \cos(\theta) + L_g \cos(\beta) \\
    r_{gy} &= L_1 \sin(\theta) - L_g \sin(\beta)
\end{align*}
\]  

Substituting for \( \sin(\beta) \) and \( \cos(\beta) \) from Equations (3) and (7) into Equations (10) and (11):

\[
\begin{align*}
    r_{gx} &= L_1 \cos(\theta) + L_g \sqrt{1 - \frac{L_1^2 \sin^2(\theta)}{L_2^2}} \\
    r_{gy} &= L_1 \sin(\theta) - L_g \frac{L_1 \sin(\theta)}{L_2}
\end{align*}
\]  

Linear velocity of the center of mass of the connecting rod, \( V_g \), in X and Y directions is obtained by differentiating Equations (12) and (13) with respect to time:

\[
\begin{align*}
    V_{gx} &= -L_1 \omega L_1 \sin(\theta) - \frac{L_g L_1^2 \omega L_1 \sin(2\theta)}{2 L_2^2 \sqrt{1 - \frac{L_1^2 \sin^2(\theta)}{L_2^2}}} \\
    V_{gy} &= L_1 \omega \cos(\theta) - \frac{L_g L_1 \omega \cos(\theta)}{L_2}
\end{align*}
\]  

Differentiating Equation (14) with respect to time will result in the linear acceleration of the center of mass of the connecting rod in the X direction, \( a_{gx} \):
\begin{align*}
a_{xy} &= -L_1 \alpha_1 \sin(\theta) - L_1 \omega_1^2 \cos(\theta) - \frac{\alpha_1 L_g L_1^2 \sin(2\theta)}{2 L_2^2 \sqrt{1 - \frac{L_1^2 \sin^2(\theta)}{L_2^2}}} \\
&- \frac{1}{L_2^2} \left(2 - \frac{2 L_1^2 \sin^2(\theta)}{L_2^2}\right) \left[\omega_1^2 L_g L_1^2 \left(2 \cos(2\theta) \sqrt{1 - \frac{L_1^2 \sin^2(\theta)}{L_2^2}} + \frac{L_1^2 \sin^2(2\theta)}{2 L_2^2 \sqrt{1 - \frac{L_1^2 \sin^2(\theta)}{L_2^2}}} \right)\right] \tag{16}
\end{align*}

For the linear acceleration of the center of mass of the connecting rod in the Y direction:

\begin{equation}
a_{xy} = L_1 \alpha_1 \cos(\theta) - L_1 \omega_1^2 \sin(\theta) - \frac{L_g L_1 \alpha_1 \cos(\theta)}{L_2} + \frac{L_g L_1 \omega_1 \sin(\theta)}{L_2} \tag{17}
\end{equation}

Considering the location of the piston as \( r_p \) with respect to the origin, the following equation could be written:

\begin{equation}
r_{px} = L_1 \cos(\theta) + L_2 \cos(\beta) \tag{18}
\end{equation}

And \( r_{py} = 0 \), since the piston does not have displacement in the Y direction.

Substituting for \( \cos(\beta) \) from Equation (7) into Equation (18):

\begin{equation}
r_{px} = L_1 \cos(\theta) + L_2 \sqrt{1 - \frac{L_1^2 \sin^2(\theta)}{L_2^2}} \tag{19}
\end{equation}

Linear velocity of the piston is obtained by differentiating Equation (19) with respect to time:

\begin{equation}
V_{px} = -L_1 \omega_1 \sin(\theta) - \frac{L_1^2 \omega_1 \sin(2\theta)}{2 L_2 \sqrt{1 - \frac{L_1^2 \sin^2(\theta)}{L_2^2}}} \tag{20}
\end{equation}

Differentiating Equation (20) with respect to time will give the linear acceleration of the piston, \( a_p \):
\[ a_{px} = -L_1 a_1 \sin(\theta) - L_1 \omega_1^2 \cos(\theta) - \frac{L_1^2 a_1 \sin(2\theta)}{2 L_2 \sqrt{1 - \frac{L_1^2 \sin(\theta)^2}{L_2^2}}} \]

\[ -\frac{1}{L_2 \left(2 - \frac{2 L_1^2 \sin(\theta)^2}{L_2^2}\right)} \left( \omega_1^2 L_1^2 \left(2 \cos(2\theta) \sqrt{1 - \frac{L_1^2 \sin(\theta)^2}{L_2^2}} + \frac{\sin(2\theta) L_1^2}{2 L_2^2 \sqrt{1 - \frac{L_1^2 \sin(\theta)^2}{L_2^2}}} \right) \right) \]  

(21)

Summing all forces acting on the piston in the X direction will result in:

\[ F_{px} = m_p a_{px} + \pi R_p^2 P_c \]  

(22)

where \( m_p \) is the mass of the piston assembly consisting of the piston, piston pin, and piston rings. \( P_c \) is the pressure in the cylinder applied on the top of the piston. \( R_p \) is the radius of the piston.

Substituting for \( a_{px} \) from Equation (21) into Equation (22), the following equation will be obtained for the force on the piston pin in the X direction, \( F_{px} \):

\[ F_{px} = -m_p \omega L_1 \omega_1 \sin(\theta) - m_p \omega L_1 \omega_1^2 \cos(\theta) - \frac{m_p L_1^2 a_1 \sin(2\theta)}{2 L_2 \sqrt{1 - \frac{L_1^2 \sin(\theta)^2}{L_2^2}}} + \pi R_p^2 P_c \]

\[ -\frac{m_p}{L_2 \left(2 - \frac{2 L_1^2 \sin(\theta)^2}{L_2^2}\right)} \left( \omega_1^2 L_1^2 \left(2 \cos(2\theta) \sqrt{1 - \frac{L_1^2 \sin(\theta)^2}{L_2^2}} + \frac{\sin(2\theta) L_1^2}{2 L_2^2 \sqrt{1 - \frac{L_1^2 \sin(\theta)^2}{L_2^2}}} \right) \right) \]  

(23)

Solving the equations of forces in X and Y direction and moment about the center of mass of the connecting rod will result in forces on the joint between the connecting rod and the crankshaft, C. These forces are given by:

\[ F_{ax} = m_r a_{rx} + F_{px} \]  

(24)
\[ F_{ay} = \frac{1}{L_2} \left( I_{zz} \alpha_2 - F_{ax} L_g - F_{px} (L_2 - L_g) \sin(\beta) \right) \cos(\beta) + m_r a_{ry} (L_2 - L_g) \] (25)

where \( I_{zz} \) and \( m_r \) are the moment of inertia and the mass of the connecting rod, respectively.

\( F_{ax} \) and \( F_{ay} \) are expressed in the global coordinate system, which is not rotating with the crankshaft. Forces expressed in a coordinate system attached to the crankshaft, better explain the loading history applied to the crankshaft. These forces are given by:

\[ F_x = F_{ax} \cos(\theta) + F_{ay} \sin(\theta) \] (26)

\[ F_y = F_{ay} \cos(\theta) - F_{ax} \sin(\theta) \] (27)

These equations have been used in a MATLAB program, detailed in Appendix B, in order to solve the mechanism for different angles of the crankshaft.
Appendix B

MATLAB program used in dynamic analysis of the slider crank mechanism developed using equations from Appendix A.

clc
clear

% measured weight of components
% piston                    330.87gr
% piston+pin                417.63gr
% connecting-rod+bolts      283.35gr
% connecting-rod            244.89
% bolts                     38.50gr
% pin                       86.79gr

l1 = 36.98494e-3;
l2 = 120.777e-3;
mcrank = 3.7191;
mrod = 283.35e-3;
l2 = 662523.4802e-9;
lg = 28.5827e-3;
mp = 417.63e-3;
load = xlsread('load.xls');

for theta_t = 1:145
    theta = (theta_t-1)*5*pi/180;
    theta_d = 2000*2*pi/60;
    theta_dd = 0;
    beta(theta_t) = asin(l1*sin(theta)/l2);
    beta_d(theta_t) = theta_d*l1*cos(theta)/l2/sqrt(1-(l1*sin(theta)/l2)^2);
    beta_dd(theta_t) = l1/l2*(theta_dd*cos(theta)-theta_d^2*sin(theta))/sqrt(1-(l1*sin(theta)/l2)^2) + theta_d^2*l1^2/l2^2*(cos(theta))^2*l1/l2*sin(theta)/((1-(l1*sin(theta)/l2)^2)^1.5);
    v_pis(theta_t) = -l1*theta_d*sin(theta) - l1^2/12*theta_d^2*sin(theta)*cos(theta)/sqrt(1-(l1/12*sin(theta)^2));
    a_rod_x(theta_t) = -l1*theta_dd*sin(theta) - l1*theta_d^2*cos(theta) - theta_dd*lg*l1^2*sin(2*theta)/(l2^2*2*sqrt(1-(l1*sin(theta)/l2)^2)) - theta_d^2*lg*l1^2/12^2*2*cos(2*theta)*sqrt(1-(l1*sin(theta)/l2)^2) +
\[
\sin(2\theta) \frac{11^2/12^2 \sin(\theta) \cos(\theta)}{\sqrt{1 - (11 \sin(\theta)/12)^2}} / (2 \cdot (1 - (11 \sin(\theta)/12)^2)) / (2 \cdot (1 - (11 \sin(\theta)/12)^2))
\]

\[
a_{\text{rod}_y}(\theta_t) = 11 \theta_{dd} \cos(\theta) - 11 \theta_d^2 \sin(\theta)
- lg \cdot 11 \text{\frac{l1}{l2}} \theta_{dd} \cos(\theta) + lg \cdot 11 \text{\frac{l1}{l2}} \theta_d^2 \sin(\theta);
\]

\[
a_{\text{pis}_x}(\theta_t) = -11 \theta_{dd} \sin(\theta) - 11^2/12 \theta_{dd} \sin(\theta) \cos(\theta)/\sqrt{1 - (11 \sin(\theta)/12)^2} - \theta_d^2 \text{\frac{l1^2}{l2}} (2 \cos(2\theta) \sqrt{1 - (11 \sin(\theta)/12)^2} + \sin(2\theta) \text{\frac{l1^2}{l2^2}} \sin(\theta) \cos(\theta))/\sqrt{1 - (11 \sin(\theta)/12)^2} / (2 \cdot (1 - (11 \sin(\theta)/12)^2));
\]

\[
f_{\text{pis}_x}(\theta_t) = mp \cdot a_{\text{pis}_x}(\theta_t) + load(\theta_t,2) \cdot 1e5 \cdot pi \cdot 0.089^2 / 4;
\]

\[
f_{\text{a}_x}(\theta_t) = m_{\text{rod}} \cdot a_{\text{rod}_x}(\theta_t) + f_{\text{pis}_x}(\theta_t);
\]

\[
f_{\text{a}_y}(\theta_t) = 1/12 * (I_{2,\text{beta}_{dd}}(\theta_t) - (f_{\text{a}_x}(\theta_t) * lg + f_{\text{pis}_x}(\theta_t) * (12 - lg)) \sin(\beta(\theta_t)))/\cos(\beta(\theta_t))) + 1/12 \cdot m_{\text{rod}} \cdot a_{\text{rod}_y}(\theta_t) * (12 - lg);
\]

\[
f_{\text{local}_x}(\theta_t) = f_{\text{a}_x}(\theta_t) \cos(\theta) + f_{\text{a}_y}(\theta_t) \sin(\theta);
\]

\[
f_{\text{local}_y}(\theta_t) = f_{\text{a}_y}(\theta_t) \cos(\theta) - f_{\text{a}_x}(\theta_t) \sin(\theta);
\]

end

figure(2)
hold off
plot(load(:,1),f_{\text{local}_x}/1000,'g--')
hold on
plot(load(:,1),f_{\text{local}_y}/1000,'-.')
plot(load(:,1),sqrt(f_{\text{local}_y}^2+f_{\text{local}_x}^2)/1000,'r-')
grid
title('Force Between Piston and Connecting rod @ 2000 rpm')
xlabel('Crankshaft Angle (Degree)')
ylabel('Force (kN)')
legend('Axial','Normal','Magnitude')